It is made up of two confocal series, one of hour-angle hyperbolas, the other of latitude ellipses. The mathematical reason for this is given in Volume III.

The azimuth is found from this diagram by:

(1) reversing the names of the latitude and declination, and when the latitude is 35°N., say, starting along the latitude ellipse from the point *K*, which is 35°S.

(2) marking the point *A* where this ellipse cuts the hyperbola corresponding to the given hour-angle.

(3) marking on the meridian the declination-point *B*, 20°S. if the declination is actually 20°N., and joining *AB*.

(4) drawing *OH* from the observer's position *O*, parallel to *BA* and cutting the horizon at *H*.

The angle *NOH* is the azimuth.

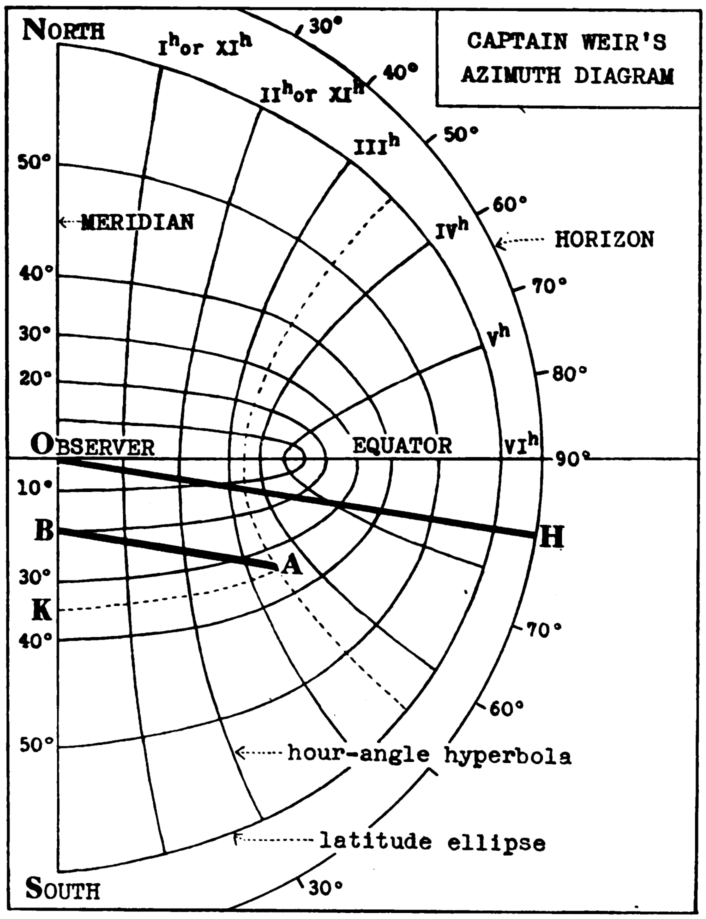

FIGURE 104.

The Latitude and Longitude of the Geographical Position. If it should happen that the altitude of the heavenly body is so large

CHAPTER XVII

WEIR'S AZIMUTH DIAGRAM

It was shown in Chapter XIV of Volume II that the azimuth of a heavenly body can be found from a diagram composed of latitude ellipses and hour-angle hyperbolas. The reason for this may be arrived at by considering the expression for the azimuth obtained by applying the four-part formula to the spherical triangle PZX.

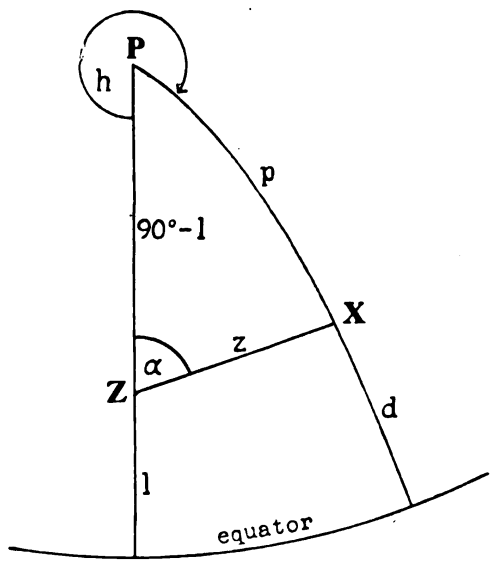

FIGURE 118.

In figure 118, the ordinary notation is adopted, and if a is the azimuth of X, the four-part formula gives:

$$\sin l \cos h = \cos l \cot p - \sin h \cot a$$

i.e.

$$\cot a = \frac{\cos l \cot p - \sin l \cos h}{\sin h}$$

$$= \frac{\tan d - \tan l \cos h}{\sec l \sin h}$$

$$= \frac{y_1 - y_2}{x_2 - x_1}$$

—where:

$$\begin{cases} x_1 = 0 \\ y_1 = \tan d \end{cases} \qquad \begin{cases} x_2 = \sec l \sin h \\ y_2 = \tan l \cos h \end{cases}$$

That is, (x_1, y_1) and (x_2, y_2) represent two points A and B in rectangular co-ordinates, the first of which lies on the y-axis at a distance tan d from the x-axis, and the second at a point determined by l and h. (Figure 119.)

When h is eliminated from the expressions for x_2 and y_2, it is seen that B lies on the ellipse :

$$\frac{x^2}{\sec^2 l}+\frac{y^2}{\tan^2 l}=1$$

When l is eliminated, it is seen that B lies on the hyperbola :

$$\frac{x^2}{\sin^2 h}-\frac{y^2}{\cos^2 h}=1$$

Hence, for any given declination, latitude and hour angle, A is a fixed point and B is the intersection of an ellipse that depends

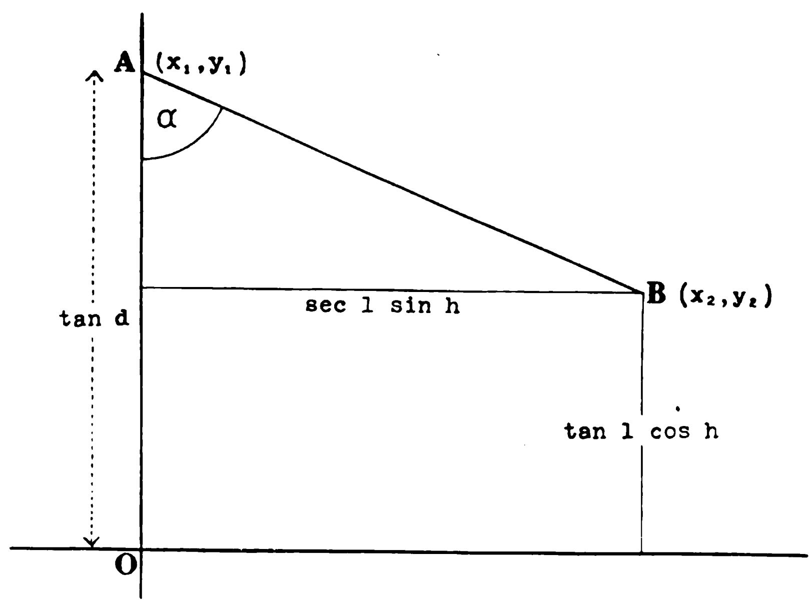

FIGURE 119.

solely on the latitude, and a hyperbola that depends solely on the hour angle. The x-axis of these two conics is the equator, and the y-axis is the observer's meridian. (Figure 120.)

If a number of ellipses and hyperbolas corresponding to different values of the latitude and hour angle are drawn, two families of conics are obtained, and it is seen that each ellipse cuts the hyperbolas at right-angles, and each hyperbola cuts the ellipses at right-angles, a fact which can be established by considering the slope of the tangents at a common point (x, y).

On the ellipse it is given by:

$$\frac{x}{\sec^2 l}+\frac{y}{\tan^2 l}\times\frac{dy}{dx}=0$$

or

$$\frac{dy}{dx}=-\frac{x\tan^2 l}{y\sec^2 l}$$

On the hyperbola it is given by:

$$\frac{x}{\sin^2 h}-\frac{y}{\cos^2 h}\times\frac{dy}{dx}=0$$

or

$$\frac{dy}{dx}=\frac{x\cos^2 h}{y\sin^2 h}$$

The product of these two values is:

$$-\frac{x^2\tan^2 l\cos^2 h}{y^2\sec^2 l\sin^2 h}$$

—and this, since x is equal to sec sin h, and y to tan l cos h, reduces to -1, thereby proving that the tangents are perpendicular. The two families of conics thus have a common focus F.

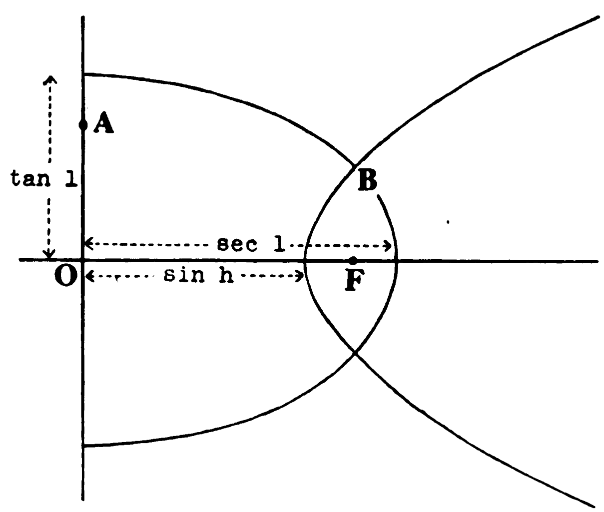

FIGURE 120.

In figure 121, the azimuth α is represented by the angle OAB, which is equal to the angle DOC, OC being parallel to AB. If, then, an azimuth circle is drawn with O as centre, the azimuth can be measured by running a parallel ruler from AB to O and noting the point C where it cuts the azimuth circle. But an adjustment is necessary in order that the convention of having the north point at the top of the azimuth circle may be followed.

In the example taken, the observer's latitude and the heavenly body's declination are both north, and the hour angle is such that α is less than 90° measured from north to east. (See figure 118.)

The north point in figure 121 is thus at the bottom of the azimuth circle, and A and B are really marked for a south latitude and a south declination equal in magnitude to the actual north latitude

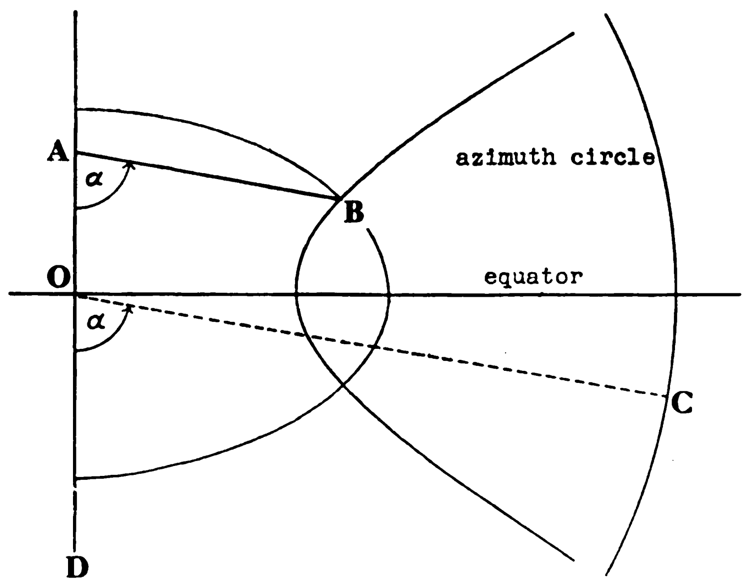

FIGURE 121.

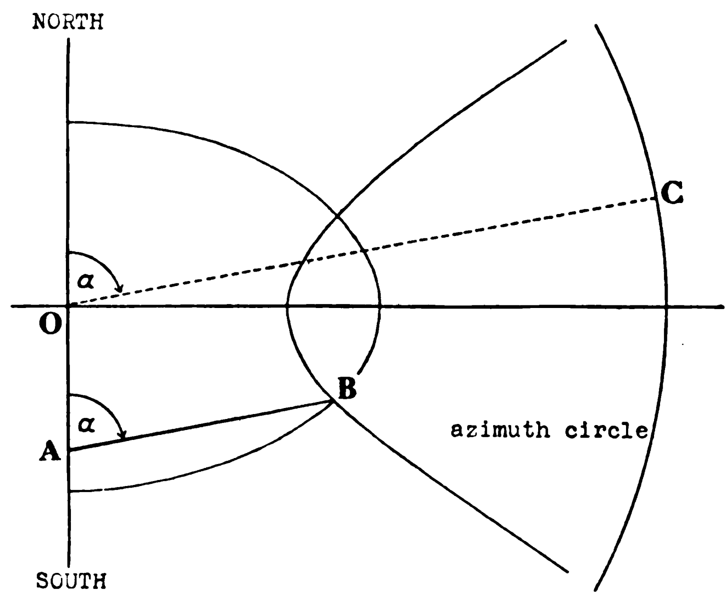

FIGURE 122.

and north declination. The same result is obtained if the convention of having the north point at the top of the azimuth circle is observed, and the names of the latitude and declination are reversed, as shown in figure 122.

The rules for using the diagram are therefore :

(1) Reverse the name of the declination and mark the point *A* on the north–south line, south of the equator if the actual declination is north, and north if it is south.
(2) Reverse the observer's latitude and follow the particular ellipse corresponding to that reversed latitude until it cuts the hyperbola corresponding to the given hour angle. This is the point *B*.
(3) Run a parallel ruler from *AB* to *O*, the observer's position, and read the azimuth on the azimuth circle at *C*.

When the hour angle gives a westerly azimuth, the same diagram can be used since the numerical value of the azimuth is the same for an hour angle of 60° as it is for one of 300°.