



$$\cos(90^\circ - d) = \cos(90^\circ - L) \cos z + \sin(90^\circ - L) \sin z \cos Z \quad (1)$$

$$\Rightarrow \cos Z = \frac{\sin d - \sin L \cos z}{\cos L \sin z} \quad (2)$$

$$\left\{ \begin{aligned} \cos z &= \cos(90^\circ - L) \cos(90^\circ - d) + \sin(90^\circ - L) \sin(90^\circ - d) \cos LHA = \\ &= \sin L \sin d + \cos L \cos d \cos LHA \end{aligned} \right. \quad (3)$$

$$\left\{ \begin{aligned} \sin z &= \sin LHA \cdot \frac{\sin(90^\circ - d)}{\sin Z} = \frac{\cos d \sin LHA}{\sin Z} \end{aligned} \right. \quad (4)$$

Insert (3) and (4) in (2)

$$\cos Z = \frac{\sin d - \sin L (\sin L \sin d + \cos L \cos d \cos LHA)}{\cos L \frac{\cos d \sin LHA}{\sin Z}} \quad (5)$$

Rearrange (5)

$$\frac{\sin Z}{\cos Z} = \frac{\cos L \cos d \sin LHA}{\sin d - \sin^2 L \sin d - \sin L \cos L \cos d \cos LHA} = \left\{ \begin{array}{l} \text{divide by} \\ \cos L \end{array} \right\} =$$

$$= \frac{\cos d \sin LHA}{\frac{\sin d}{\cos L} - (1 - \cos^2 L) \frac{\sin d}{\cos L} - \sin L \cos d \cos LHA}$$

$$= \frac{\cos d \sin LHA}{\cos L \sin d - \sin L \cos d \cos LHA} = \tan Z$$