Notes on Cloudy Weather Johnson's Double Chronometer Method

Version 1

In the 19th century a mariner typically determined longitude by a morning time sight and latitude by a noon sight. If the weather was cloudy at either time he became uncertain of his position. Following Sumner's discovery it was known that latitude/longitude could be determind by double chronometer sights. The method involved taking two time sights, reducing each with two assumed latitudes and then plotting two LOPs. There were problems with this method - the amount of computation required and the mess created on the chart by plotting.

In about 1874 Alfred Chalice Johnson RN (AKA cloudy weather Johnson) published his booklet "On Finding Latitude and Longitude in Cloudy Weather and at Other Times". This described a double chronometer method which determined latitude/longitude by calculation rather that by plotting. A rough sketch was required to determine the sign of the latitude and longitude corrections. Copies were supplied to HM ships by Admiralty order. The 28th edition, of which I have a copy, was published in 1905 - it was clearly a popular volume. In 1905 the volume could be purchased for five shillings - about $NZ46 (24GBP or $US30) in 2020 money. If a late edition came up for sale in 2020 the asking price would probably be about $US15 - $US30.

A footnote in Johnson 28th edition 1905 (page 35) states:

> This table (referring to table II) was first published in the 4th edition of this little book in 1874. It was subsequently, by the Author's permission, inserted in Lecky's 'Wrinkles", Inmans Nautical Tables &c. In the former, and in the General Utility tables, by the same Author, it now appears in an expanded form, as Table (C).

A footnote in Burton 1947 (page 13) states:

> The method introduced by Mr AC Johnson RN of "Cloudy Weather" fame. It was he who originated Table C. The A and B table is much older, having been first published anonymously in the Nautical Magazine in 1846.

Lecky had positive things to say about the method. In the 1884 edition of his work (p321) he compared Johnson's method with the chord (two-point) Sumner, and wrote:

> But the Double Altitude problem, when worked in full, according to Sumner, is a formidable affair, and the rules at the finish are so complicated as to scare most ordinary seafaring men.

and later

> The easier method recently introduced by A C Johnson M.A. Naval Instructor on board her Majesty's training ship "Brittania" is therefore a most welcome omprovement. It is short, easily understood, and accurate, and there is little in the calculation with which the navigator is not already familiar.

The above is a simplification of the development of the ABC tables and the double altitude method. In his book *A History of Nautical Astronomy* Cotter discusses the subject in detail.

Johnson's Double Chronometer method uses plane trigonometry and the ABC tables, where

A = cot HA Tan lat
B = cosec HA tan dec
C + cot Az sec lat
and A + B = C.

Worked Example

I took a series of AM and PM time sights and reduced them by Martelli. I then chose two sights which (by courtesy of the Clockwork Mapping anti-spoof app) I knew were reasonably accurate. I calculated C from both Norie's ABC tables and also from Johnson's table I to confirm that the tables are the same. Johnson's table I is Norie's tables A and B combined. Johnson's table II is Norie's table C.

Johnson gives a method for using his tables I and II for multiplication and division but I cheated and used a calculator.

AM sight
========

lat 41 S
dec 4 46.8 S
t 44 09 E
long 174 59.6 E

A 0.90 N
B 0.12 S

Subtract by rule (different names) gives C = 0.78 N
From which azimuth = N59.5E.

PM Sight
=======

lat 41 S
dec 4 30.3 S
t 59 07 W
long = 175 5.9 E

A 0.52 N
B 0.09 S

Subtract by rule (different names) gives c = 0.43
From which azimuth = N72W

Calculate Position
=============

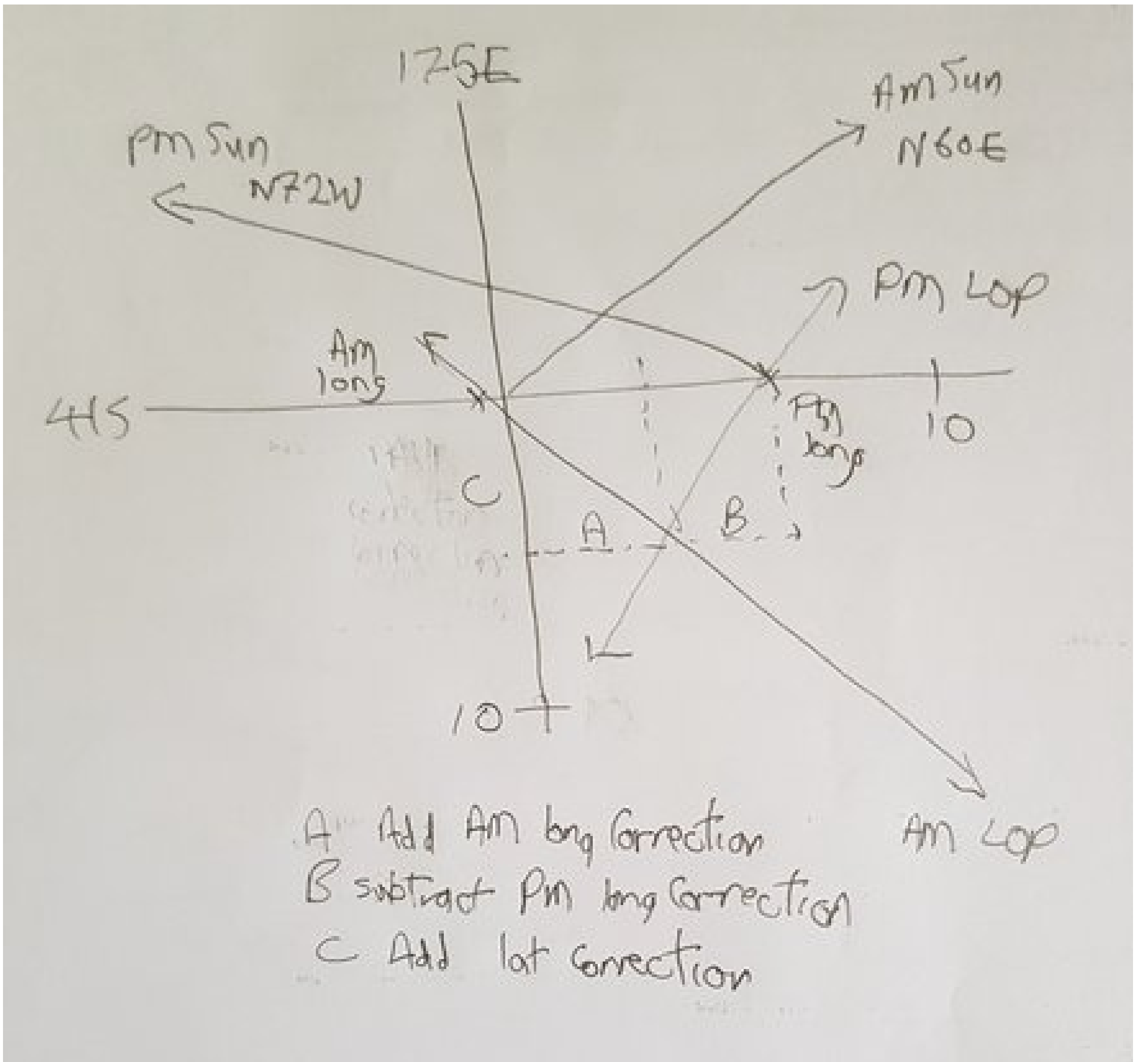
lat correction = diff long/ (sum or diff C values)

The observations are in adjacent quadrants so add the C values.

lat correction = 6.3 / 1.21 = 5.2'

long correction am sight = lat correction * am C = 5.2 * 0.78 = 4.1'
long correction pm sight = lat correction * pm C = 5.2 * 0.43 = 2.2'

Using the calculated azimuths draw a rough sketch to determine the signs of the corrections. In his instructions on this method Burton points out that the position can be alternatively determined by plotting the rough sketch to scale (1947 page 14).



Observed lat = 41 00 + 00 05.2 = 41 05.2' S

Observed am long = 174 59.6 + 4.1' = 175 3.7' E
Observed pm long = 175 5.9 - 2.2' = 175 3.7' E

I did not feel the Pacific plate move between the AM and PM sights so the two longitudes should be equal.

My GNSS position is 41 06.5' S
175 05.2' E

Burton's Instructions

The following instructions for Johnson's Double Chronometer method are are from the 1947 edition of Burton's Nautical tables.

rection, for which purpose

Table C is also used for combining two simultaneous or successive "chronometer" sights in order to obtain a fix.* For this purpose the process is as follows :—

Calculate the two longitudes by any "chronometer" formula, and take out C for each sight. (If the sights have been successively obtained, the first sight is brought up to the time of the second sight by means of the Traverse Table).

Then—

$$\frac{\text{Difference between longitudes}}{\text{Sum or difference of C values}} = \text{Correction for latitude.}$$

Rule for C's—

Observations in ADJACENT quadrants, use SUM.
" SAME or OPPOSITE quadrant(s), use DIFFERENCE.

The latitude correction being known, each C is multiplied by this amount to obtain the respective longitude corrections. A rough sketch of the position lines is then made in order to show how these corrections are to be applied.

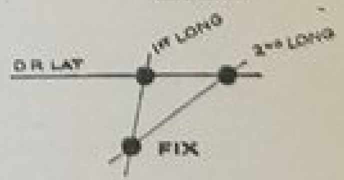
Example :

Successive observations, same quadrant.

1st Obs. 08.16	Lat. D.R.	33° 27′.0 N.	Long. Obs.	26° 13′.5 E.	⊙ Bg. S83°E.	C = .15
Run S72°E. 36 M.		11′.1 S.		41′.0 E.		
1st Obs. 11.00	Lat. D.R.	33° 15′.9 N.	Long. Obs.	26° 54′.5 E.		
2nd Obs. 11.00	Lat. D.R.	33° 15′.9 N.	Long. Obs.	26° 59′.75 E.	⊙ Bg. S38½°E.	C = 1.50
				Diff. 5′.25		Diff. 1.35

$\frac{5.25}{1.35} = 3.9 =$ Lat. Corr.

$3.9 \times .15 = .595 =$ Corr. for 1st Long.
$3.9 \times 1.50 = 5.85 =$ Corr. for 2nd Long.



11.00 D.R. Lat. 33° 15′.9 N.	1st Long. Obs. 26° 54′.5 E.	2nd Long. Obs. 26° 59′.75 E.
Corr. 3.9 S.	Corr. 0.6 W.	Corr. 5.85 W.
11.00 True Lat. 33° 12.0 N.	True Long. 26° 53.9 E.	True Long. 26° 53.9 E.

*The method introduced by Mr. A. C. Johnson, R.N., of "Cloudy Weather" fame. It was he who originated Table C. The A and B Table is much older, having been first published anonymously in the Nautical Magazine in 1846.

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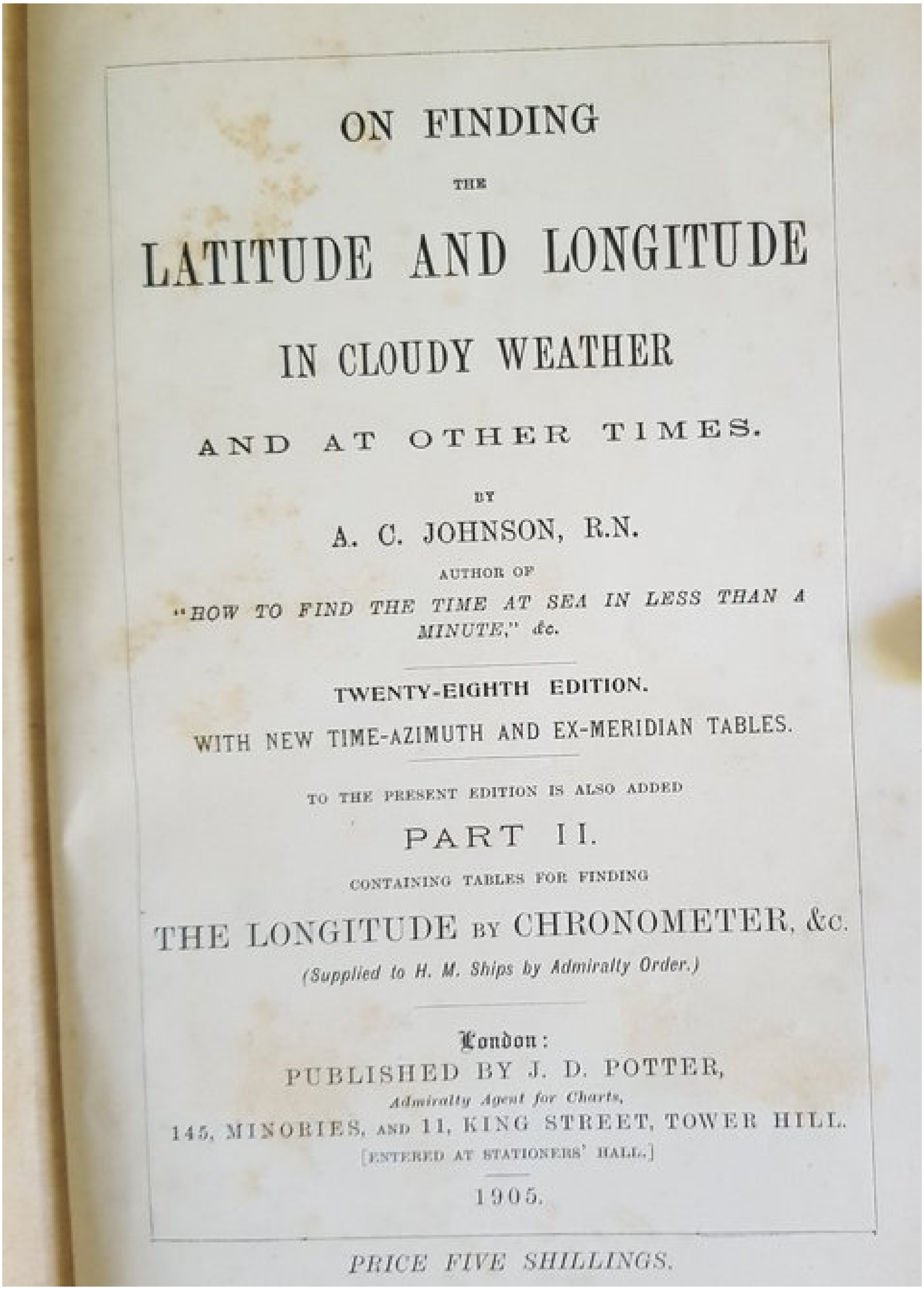
Extracts From ***On Finding the Latitude and Longitude In Cloudy Weather***



ON FINDING

THE

LATITUDE AND LONGITUDE

IN CLOUDY WEATHER

AND AT OTHER TIMES.

BY

A. C. JOHNSON, R.N.

AUTHOR OF

"HOW TO FIND THE TIME AT SEA IN LESS THAN A MINUTE," &c.

TWENTY-EIGHTH EDITION.

WITH NEW TIME-AZIMUTH AND EX-MERIDIAN TABLES.

TO THE PRESENT EDITION IS ALSO ADDED

PART II.

CONTAINING TABLES FOR FINDING

THE LONGITUDE BY CHRONOMETER, &c.

(Supplied to H. M. Ships by Admiralty Order.)

London:

PUBLISHED BY J. D. POTTER,

Admiralty Agent for Charts,

145, MINORIES, AND 11, KING STREET, TOWER HILL.

[ENTERED AT STATIONERS' HALL.]

1905.

PRICE FIVE SHILLINGS.

A Fine Victorian Gentleman

I do not know Johnson's dates of birth and death. He may also have been an Edwardian. If born before 1837 he would have been a subject of William IV.

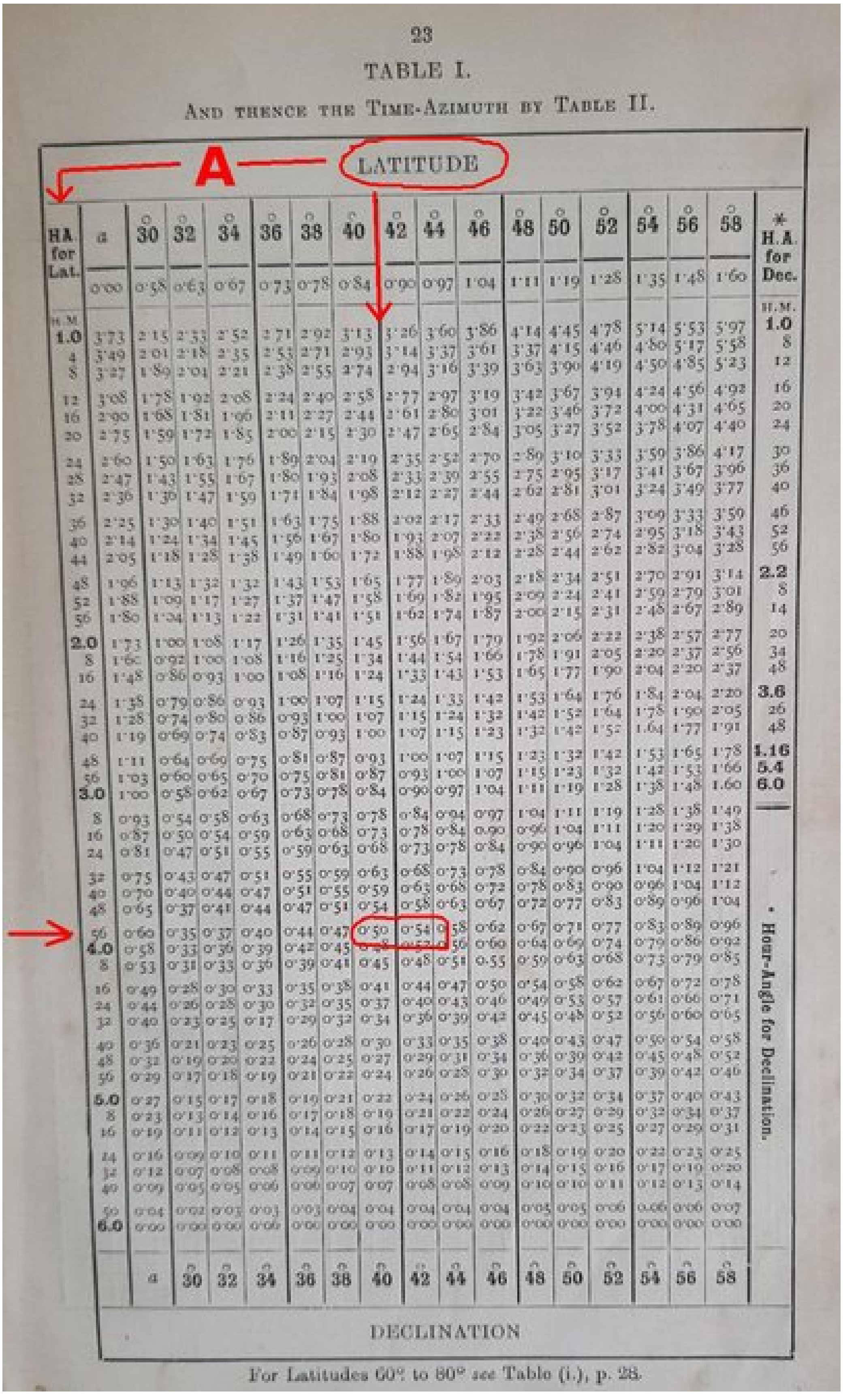


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TABLE I.

AND THENCE THE TIME-AZIMUTH BY TABLE II.

A — LATITUDE

HA. for Lat.	a	30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50°	52°	54°	56°	58°	* H.A. for Dec.
	0·00	0·58	0·63	0·67	0·73	0·78	0·84	0·90	0·97	1·04	1·11	1·19	1·28	1·35	1·48	1·60	
H. M.																	H. M.
1.0	3·73	2·15	2·33	2·52	2·71	2·92	3·13	3·26	3·60	3·86	4·14	4·45	4·78	5·14	5·53	5·97	1.0
4	3·49	2·01	2·18	2·35	2·53	2·71	2·93	3·14	3·37	3·61	3·37	4·15	4·46	4·80	5·17	5·58	8
8	3·27	1·89	2·04	2·21	2·38	2·55	2·74	2·94	3·16	3·39	3·63	3·90	4·19	4·50	4·85	5·23	12
12	3·08	1·78	1·92	2·08	2·24	2·40	2·58	2·77	2·97	3·19	3·42	3·67	3·94	4·24	4·56	4·92	16
16	2·90	1·68	1·81	1·96	2·11	2·27	2·44	2·61	2·80	3·01	3·22	3·46	3·72	4·00	4·31	4·65	20
20	2·75	1·59	1·72	1·85	2·00	2·15	2·30	2·47	2·65	2·84	3·05	3·27	3·52	3·78	4·07	4·40	24
24	2·60	1·50	1·63	1·76	1·89	2·04	2·19	2·35	2·52	2·70	2·89	3·10	3·33	3·59	3·86	4·17	30
28	2·47	1·43	1·55	1·67	1·80	1·93	2·08	2·33	2·39	2·55	2·75	2·95	3·17	3·41	3·67	3·96	36
32	2·36	1·36	1·47	1·59	1·71	1·84	1·98	2·12	2·27	2·44	2·62	2·81	3·01	3·24	3·49	3·77	40
36	2·25	1·30	1·40	1·51	1·63	1·75	1·88	2·02	2·17	2·33	2·49	2·68	2·87	3·09	3·33	3·59	46
40	2·14	1·24	1·34	1·45	1·56	1·67	1·80	1·93	2·07	2·22	2·38	2·56	2·74	2·95	3·18	3·43	52
44	2·05	1·18	1·28	1·38	1·49	1·60	1·72	1·88	1·98	2·12	2·28	2·44	2·62	2·82	3·04	3·28	56
48	1·96	1·13	1·32	1·32	1·43	1·53	1·65	1·77	1·89	2·03	2·18	2·34	2·51	2·70	2·91	3·14	2.2
52	1·88	1·09	1·17	1·27	1·37	1·47	1·58	1·69	1·82	1·95	2·09	2·24	2·41	2·59	2·79	3·01	8
56	1·80	1·04	1·13	1·22	1·31	1·41	1·51	1·62	1·74	1·87	2·00	2·15	2·31	2·48	2·67	2·89	14
2.0	1·73	1·00	1·08	1·17	1·26	1·35	1·45	1·56	1·67	1·79	1·92	2·06	2·22	2·38	2·57	2·77	20
8	1·60	0·92	1·00	1·08	1·16	1·25	1·34	1·44	1·54	1·66	1·78	1·91	2·05	2·20	2·37	2·56	34
16	1·48	0·86	0·93	1·00	1·08	1·16	1·24	1·33	1·43	1·53	1·65	1·77	1·90	2·04	2·20	2·37	48
24	1·38	0·79	0·86	0·93	1·00	1·07	1·15	1·24	1·33	1·42	1·53	1·64	1·76	1·84	2·04	2·20	3.6
32	1·28	0·74	0·80	0·86	0·93	1·00	1·07	1·15	1·24	1·32	1·42	1·52	1·64	1·78	1·90	2·05	26
40	1·19	0·69	0·74	0·83	0·87	0·93	1·00	1·07	1·15	1·23	1·32	1·42	1·52	1.64	1·77	1·91	48
48	1·11	0·64	0·69	0·75	0·81	0·87	0·93	1·00	1·07	1·15	1·23	1·32	1·42	1·53	1·65	1·78	1.16
56	1·03	0·60	0·65	0·70	0·75	0·81	0·87	0·93	1·00	1·07	1·15	1·23	1·32	1·42	1·53	1·66	5.4
3.0	1·00	0·58	0·62	0·67	0·73	0·78	0·84	0·90	0·97	1·04	1·11	1·19	1·28	1·38	1·48	1.60	6.0
8	0·93	0·54	0·58	0·63	0·68	0·73	0·78	0·84	0·94	0·97	1·04	1·11	1·19	1·28	1·38	1·49	
16	0·87	0·50	0·54	0·59	0·63	0·68	0·73	0·78	0·84	0.90	0·96	1·04	1·11	1·20	1·29	1·38	
24	0·81	0·47	0·51	0·55	0·59	0·63	0·68	0·73	0·78	0·84	0·90	0·96	1·04	1·11	1·20	1·30	
32	0·75	0·43	0·47	0·51	0·55	0·59	0·63	0·68	0·73	0·78	0·84	0·90	0·96	1·04	1·12	1·21	
40	0·70	0·40	0·44	0·47	0·51	0·55	0·59	0·63	0·68	0·72	0·78	0·83	0·90	0·96	1·04	1·12	
48	0·65	0·37	0·41	0·44	0·47	0·51	0·54	0·58	0·63	0·67	0·72	0·77	0·83	0·89	0·96	1·04	
56	0·60	0·35	0·37	0·40	0·44	0·47	0·50	0·54	0·58	0·62	0·67	0·71	0·77	0·83	0·89	0·96	
4.0	0·58	0·33	0·36	0·39	0·42	0·45	[illegible]	[illegible]	0·56	0·60	0·64	0·69	0·74	0·79	0·86	0·92	
8	0·53	0·31	0·33	0·36	0·39	0·41	0·45	0·48	0·51	0.55	0·59	0·63	0·68	0·73	0·79	0·85	
16	0·49	0·28	0·30	0·33	0·35	0·38	0·41	0·44	0·47	0·50	0·54	0·58	0·62	0·67	0·72	0·78	
24	0·44	0·26	0·28	0·30	0·32	0·35	0·37	0·40	0·43	0·46	0·49	0·53	0·57	0·61	0·66	0·71	
32	0·40	0·23	0·25	0·17	0·29	0·32	0·34	0·36	0·39	0·42	0·45	0·48	0·52	0·56	0·60	0·65	
40	0·36	0·21	0·23	0·25	0·26	0·28	0·30	0·33	0·35	0·38	0·40	0·43	0·47	0·50	0·54	0·58	
48	0·32	0·19	0·20	0·22	0·24	0·25	0·27	0·29	0·31	0·34	0·36	0·39	0·42	0·45	0·48	0·52	
56	0·29	0·17	0·18	0·19	0·21	0·22	0·24	0·26	0·28	0·30	0·32	0·34	0·37	0·39	0·42	0·46	
5.0	0·27	0·15	0·17	0·18	0·19	0·21	0·22	0·24	0·26	0·28	0·30	0·32	0·34	0·37	0·40	0·43	
8	0·23	0·13	0·14	0·16	0·17	0·18	0·19	0·21	0·22	0·24	0·26	0·27	0·29	0·32	0·34	0·37	
16	0·19	0·11	0·12	0·13	0·14	0·15	0·16	0·17	0·19	0·20	0·22	0·23	0·25	0·27	0·29	0·31	
24	0·16	0·09	0·10	0·11	0·11	0·12	0·13	0·14	0·15	0·16	0·18	0·19	0·20	0·22	0·23	0·25	
32	0·12	0·07	0·08	0·08	0·09	0·10	0·10	0·11	0·12	0·13	0·14	0·15	0·16	0·17	0·19	0·20	
40	0·09	0·05	0·05	0·06	0·06	0·07	0·07	0·08	0·08	0·09	0·10	0·10	0·11	0·12	0·13	0·14	
50	0·04	0·02	0·03	0·03	0·03	0·04	0·04	0·04	0·04	0·04	0·05	0·05	0·06	0.06	0·06	0·07	
6.0	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	
	a	30°	32°	34°	36°	38°	40°	42°	44°	46°	48°	50°	52°	54°	56°	58°	

DECLINATION

* Hour-Angle for Declination.

For Latitudes 60° to 80° see Table (i.), p. 28.

t = 59 deg 07' = 3 hr 56 min 28 sec.
Lat 41S   Dec 4 deg 30.3'

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TABLE I.

For the Longitude Correction,

LATITUDE

HA. for Lat.	a	0°	2°	4°	6°	8°	10°	12°	14°	16°	18°	20°	22°	24°	26°	28°	* HA. for Dec.
	0·00	0·00	0·03	0·07	0·10	0·14	0·18	0·21	0·25	0·29	0·33	0·36	0·40	0·45	0·49	0·53	
H. M.																	H. M.
1.0	3·73	0·00	0·13	0·26	0·39	0·52	0·66	0·79	0·93	1·07	1·21	1·36	1·51	1·66	1·83	1·98	1.0
4	3·49	0·00	0·12	0·24	0·37	0·49	0·61	0·74	0·87	1·00	1·13	1·27	1·41	1·55	1·70	1·85	8
8	3·27	0·00	0·11	0·23	0·34	0·46	0·58	0·69	0·82	0·94	1·06	1·19	1·32	1·46	1·59	1·74	12
12	3·08	0·00	0·11	0·22	0·32	0·43	0·54	0·65	0·77	0·88	1·00	1·12	1·24	1·37	1·50	1·64	16
16	2·90	0·00	0·10	0·20	0·30	0·41	0·51	0·62	0·72	0·83	0·94	1·06	1·17	1·29	1·42	1·54	20
20	2·75	0·00	0·09	0·19	0·28	0·39	0·48	0·58	0·68	0·79	0·89	1·00	1·11	1·22	1·31	1·46	24
24	2·60	0·00	0·09	0·18	0·27	0·37	0·46	0·55	0·65	0·75	0·85	0·95	1·08	1·16	1·27	1·38	30
28	2·47	0·00	0·08	0·17	0·26	0·35	0·44	0·53	0·62	0·71	0·80	0·90	1·00	1·10	1·21	1·32	36
32	2·36	0·00	0·08	0·16	0·25	0·33	0·41	0·50	0·59	0·68	0·76	0·85	0·95	1·05	1·15	1·25	40
36	2·25	0·00	0·08	0·16	0·24	0·32	0·40	0·48	0·56	0·64	0·73	0·81	0.91	1·00	1·09	1·19	46
40	2·14	0·00	0·07	0·15	0·22	0·30	0·38	0·46	0·53	0·61	0·70	0·78	0·87	0·95	1·05	1·14	52
44	2·05	0·00	0·07	0·14	0·21	0·29	0·36	0·44	0·51	0·59	0·67	0·75	0·83	0·91	1·00	1·09	56
48	1·96	0·00	0·07	0·14	0·21	0·28	0·35	0·42	0·49	0·56	0·64	0·71	0·79	0·87	0·96	1·04	2.2
52	1·88	0·00	0·06	0·13	0·20	0·26	0·33	0·40	0·47	0·54	0·61	0·68	0·76	0·84	0·92	1·00	8
56	1·80	0·00	0·06	0·13	0·19	0·25	0·32	0·38	0·45	0·52	0·59	0·66	0·73	0·80	0·88	0·96	14
2.0	1·73	0·00	0·06	0·12	0·18	0·24	0·30	0·37	0·43	0·50	0·58	0·63	0·70	0·77	0·84	0·92	20
8	1·60	0·00	0·06	0·11	0·17	0·22	0·28	0·35	0·40	0·46	0·52	0·58	0·65	0·71	0·78	0·85	34
16	1·48	0·00	0·05	0·10	0·16	0·21	0·26	0·31	0·37	0·42	0·48	0·54	0·60	0·66	0·72	0·79	48
24	1·38	0·00	0·05	0·10	0·14	0·19	0·24	0·29	0·34	0·39	0·45	0·50	0·56	0.61	0·67	0·73	3.6
32	1·28	0·00	[illegible]	[illegible]	[illegible]	0·18	0·23	0·27	0·32	0·37	0·42	0·47	0·52	0·57	0·62	0·68	26
40	1·19	0·00	0·04	0·08	0·12	0·17	0·21	0·25	0·30	0·34	0·39	0·43	0·48	0·53	0·55	0·63	48
48	1·11	0·00	0.04	0·08	0·12	0·16	0·20	0·24	0·28	0·32	0·36	0·40	0·45	0·49	0·54	0·59	4.16
56	1·03	0·00	0·04	0·07	0·11	0·15	0·19	0·22	0·26	0·30	0·34	0·38	0·42	0·46	0·50	0·55	5.4
3.0	1·00	0·00	0·04	0·07	0·10	0·14	0·18	0·21	0·25	0·29	0·32	0·36	0·40	0·44	0·49	0·53	6.0
8	0·93	0·00	0·03	0·06	0·10	0·13	0·16	0·19	0·23	0·27	0·30	0.34	0·38	0·41	0·45	0·50	
16	0·87	0·00	0·03	0·06	0·09	0·12	0·15	0·18	0·22	0·25	0·28	0·32	0·35	0·39	0·42	0·46	
24	0·81	0·00	0·03	0·06	0·08	0·11	0·14	0·17	0·20	0·23	0·26	0·29	0·33	0·36	0·39	0·43	
32	0·75	0·00	0·03	0·05	0·08	0·11	0·13	0·16	0·19	0·22	0·24	0·27	0·30	0·34	0·37	0·40	
40	0·70	0·00	0·02	0·05	0·07	0·10	0·12	0·15	0·17	0·20	0·23	0·25	0·28	0·31	0·34	0·37	
48	0·65	0·00	0·02	0·04	0·07	0·09	0·11	0·14	0·16	0·19	0·21	0·24	0·26	0·29	0·32	0·34	
56	0·60	0·00	0·02	0·04	0·06	0·08	0·10	0·13	0·15	0·17	0·19	0·22	0·24	0·27	0·29	0·32	
4.0	0·58	0·00	0·02	0·04	0·06	0·08	0·10	0·12	0·14	0·17	0·19	0·21	0·23	0·26	0·28	0·31	
8	0·53	0·00	0·02	0·04	0·06	0·07	0·09	0·11	0·13	0·15	0·17	0·19	0·21	0·24	0·26	0·28	
16	0·49	0·00	0·02	0·03	0·05	0·07	0·09	0·10	0·12	0·14	0·16	0·18	0·20	0·22	0·24	0·26	
24	0·44	0·00	0·02	0·03	0·05	0·06	0·08	0·09	0·11	0·13	0·14	0·16	0·18	0·20	0·22	0·24	
32	0·40	0·00	0·01	0·03	0·04	0·06	0·07	0·09	0·10	0·12	0·13	0·15	0·16	0·18	0·20	0·21	
40	0·36	0·00	0·01	0·03	0·04	0·05	0·06	0·08	0·09	0·10	0·12	0·13	0·15	0·16	0·18	0·19	
48	0·32	0·00	0·01	0·02	0·03	0·05	0·06	0·07	0·08	0·09	0·11	0·12	0·13	0·14	0·16	0·17	
56	0·29	0·00	0·01	0·02	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·10	0·12	0·13	0·14	0·15	
5.0	0·27	0·00	0·01	0·01	0·03	0·04	0·05	0·06	0·07	0·08	0·09	0·10	0·11	0·12	0·13	0·14	
8	0·23	0·00	0·01	0.01	0·02	0·03	0·04	0·05	0·06	0·07	0·07	0·08	0·09	0·10	0·11	0·12	
16	0·19	0·00	0·01	0·01	0·02	0·03	0·03	0·04	0·05	0·06	0·06	0·07	0·08	0·09	0·09	0·10	
24	0·16	0·00	0·01	0·01	0·02	0·03	0·03	0·03	0·04	0·04	0·05	0·06	0·06	0·07	0·08	0·08	
32	0·12	0·00	0·01	0·01	0·01	0·02	0·02	0·03	0·03	0·03	0·04	0·04	0·05	0·05	0·06	0·06	
40	0·09	0·00	0.01	0·01	0·01	0·01	0·02	0·02	0·02	0·02	0·03	0·03	0·03	0·04	0·04	0·05	
56	0·04	0·00	0·00	0·00	0·00	0·00	0·01	0·01	0·01	0·01	0·01	0·02	0·02	0·02	0·02	0·02	
6.0	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	
		0°	2°	4°	6°	8°	10°	12°	14°	16°	18°	20°	22°	24°	26°	28°	

* DECLINATION

B

* Hour-Angle for Declination

For Latitudes 60° to 80°, see Table (i.), p. 28.

t = 59 deg 07' = 3 hr 56 min 28 sec.

Lat 41S Dec 4 deg 30.3'

C

TABLE II.

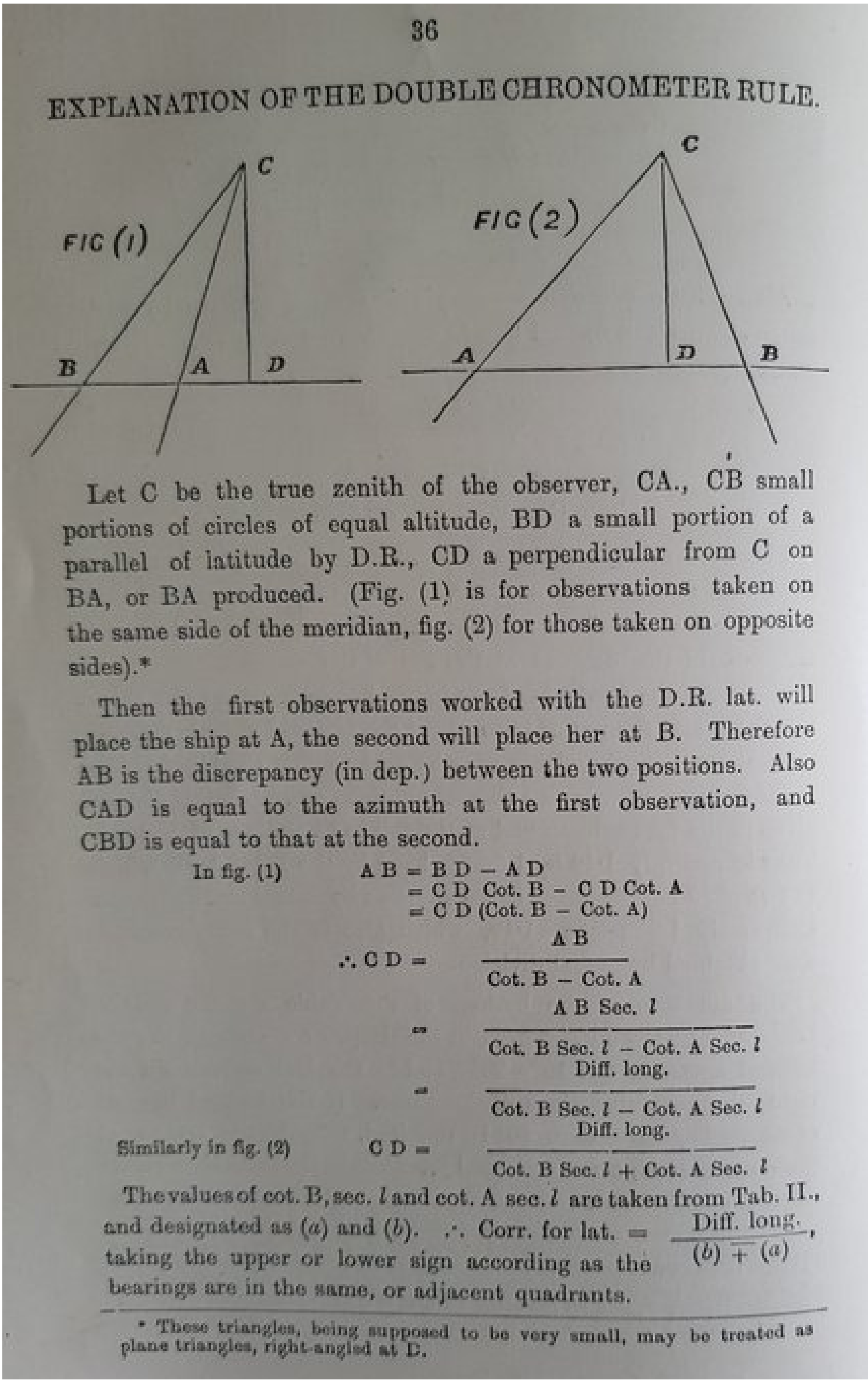
Time-Azimuth, Alt.-Azimuth, Great-Circle Sailing, &c.

Bearing.	LATITUDE.															Equivalent Bearing for Error in Alt.	For the Alt.-Azimuth
	0°	34°	36°	38°	40°	42°	44°	46°	48°	50°	52°	54°	56°	58°	60°		
	a	1·21	1·24	1·27	1·31	1·35	1·39	1·44	1·49	1·56	1·62	1·70	1·79	1·89	2·00		
10°	5·67	6·84	7·01	7·20	7·40	7·63	7·88	8·16	8·48	8·82	9·21	9·65	10·14	10·70	11·33	10°	·98
12	4·71	5·67	5·81	5·97	6·14	6·33	6·54	6·77	7·03	7·32	7·64	8·00	8·41	8·88	9·41	12	·98
14	4·01	4·84	4·95	5·09	5·23	5·40	5·58	5·77	5·99	6·24	6·51	6·82	7·17	7·57	8·02	14	·97
16	3·49	4·21	4·31	4·43	4·55	4·69	4·85	5·02	5·21	5·42	5·66	5·93	6·24	6·58	6·97	15	·96
18	3·08	3·71	3·80	3·90	4·02	4·14	4·28	4·43	4·60	4·79	5·00	5·24	5·50	5·81	6·15	17	·95
20	2·75	3·31	3·39	3·49	3·59	3·70	3·82	3·95	4·11	4·27	4·46	4·67	4·91	5·19	5·49	19	·94
22	2·47	2·98	3·06	3·14	3·23	3·33	3·44	3·56	3·70	3·85	4·02	4·21	4·43	4·67	4·95	21	·93
24	2·25	2·71	2·77	2·85	2·93	3·02	3·12	3·23	3·36	3·49	3·65	3·82	4·02	4·24	4·49	22	·91
26	2·05	2·47	2·53	2·60	2·68	2·76	2·85	2·95	3·06	3·19	3·33	3·49	3·66	3·87	4·10	24	·90
28	1·88	2·27	2·32	2·39	2·45	2·53	2·61	2·71	2·81	2·92	3·05	3·20	3·36	3·55	3·76	25	·88
30	1·73	2·09	2·14	2·20	2·26	2·33	2·41	2·49	2·60	2·69	2·81	2·95	3·10	3·27	3·46	26	·87
32	1·60	1·93	1·98	2·03	2·09	2·15	2·22	2·30	2·39	2·49	2·60	2·72	2·86	3·02	3·20	28	·85
34	1·48	1·79	1·83	1·88	1·93	1·99	2·06	2·13	2·22	2·31	2·41	2·52	2·65	2·80	2·96	29	·83
36	1·38	1·66	1·70	1·74	1·80	1·85	1·91	1·98	2·06	2·14	2·24	2·34	2·46	2·60	2·75	30	·81
38	1·28	1·54	1·58	1·62	1·67	1·72	1·78	1·84	1·91	1·99	2·08	2·18	2·29	2·41	2·56	32	·79
40	1·19	1·44	1·47	1·51	1·55	1·60	1·66	1·72	1·78	1·85	1·94	2·03	2·13	2·25	2·38	33	·77
42	1·11	1·34	1·37	1·41	1·45	1·49	1·54	1·60	1·66	1·73	1·80	1·89	1·99	2·09	2·22	34	·74
44	1·04	1·25	1·28	1·31	1·35	1·39	1·44	1·49	1·55	1·61	1·68	1·76	1·85	1·95	2·07	35	·72
46	0·97	1·16	1·19	1·23	1·26	1·30	1·34	1·39	1·44	1·50	1·56	1·64	1·73	1·82	1·93	36	·69
48	0·90	1·09	1·11	1·14	1·17	1·21	1·25	1·30	1·35	1·40	1·46	1·53	1·61	1·70	1·80	37	·67
50	0·84	1·01	1·04	1·06	1·09	1·13	1·16	1·21	1·25	1·31	1·36	1·43	1·50	1·58	1·68	38	·64
52	0·78	0·94	0·96	0·99	1·01	1·05	1·09	1·12	1·17	1·22	1·27	1·33	1·40	1·47	1·56	38	·62
54	0·73	0·88	0·90	0·92	0·95	0·98	1·01	1·04	1·09	1·13	1·18	1·23	1·30	1·37	1·45	39	·59
56	0·67	0·81	0·83	0·85	0·88	0·91	0·94	0·97	1·01	1·05	1·10	1·15	1·21	1·27	1·35	40	·56
58	0·63	0·75	0·77	0·79	0·81	0·84	0·87	0·90	0·93	0·97	1·01	1·06	1·12	1·18	1·25	40	·53
60	0·58	0·70	0·71	0·73	0·75	0·78	0·80	0·83	0·86	0·90	0·94	0·98	1·03	1·09	1·15	41	·50
62	0·53	0·64	0·66	0·67	0·69	0·72	0·74	0·76	0·79	0·83	0·86	0·90	0·95	1·00	1·06	41	·47
64	0·49	0·59	0·60	0·62	0·64	0·66	0·68	0·70	0·73	0·76	0·79	0·83	0·87	0·92	0·97	42	·44
66	0·45	0·54	0·55	0·56	0·58	0·60	0·62	0·64	0·66	0·69	0·72	0·76	0·79	0·84	0·89	42	·41
68	0·40	0·49	0·50	0·51	0·53	0·54	0·56	0·58	0·60	0·63	0·65	0·69	0·72	0·76	0·81	43	·37
70	0·36	0·44	0·45	0·46	0·47	0·49	0·51	0·52	0·54	0·57	0·59	0·62	0·65	0·68	0·73	43	·34
72	0·33	0·39	0·40	0·41	0·42	0·44	0·45	0·47	0·49	0·51	0·53	0·55	0·58	0·61	0·65	44	·31
74	0·29	0·34	0·36	[illegible]	[illegible]	[illegible]	0·40	0·41	0·43	0·44	0·46	0·49	0·52	0·54	0·57	44	·28
76	0·25	0·30	0·31	0·31	0·32	0·33	0·34	0·36	0·37	0·39	0·40	0·42	0·45	0·47	0·50	44	·24
78	0·21	0·25	0·26	0·27	0·28	0·29	0·29	0·30	0·32	0·33	0·34	0·36	0·38	0·40	0·42	44	·21
80	0·18	0·21	0·22	0·22	0·23	0·24	0·24	0·25	0·26	0·27	0·29	0·30	0·31	0·33	0·35	44	·17
82	0·14	0·17	0·17	0·18	0·18	0·19	0·19	0·20	0·21	0·22	0·23	0·24	0·25	0·26	0·28	45	·14
84	0·10	0·13	0·13	0·13	0·14	0·14	0·14	0·15	0·16	0·16	0·17	0·18	0·19	0·20	0·21	45	·10
86	0·07	0·08	0·08	0·09	0·09	0·09	0·10	0·10	0·10	0·11	0·11	0·12	0·12	0·13	0·14	45	·07
88	0·03	0·04	0·04	0·04	0·04	0·05	0·05	0·05	0·05	0·05	0·06	0·06	0·06	0·07	0·07	45	·03
89	0·01	0·01	0·01	0·01	0·01	0·01	0·01	0·01	0·02	0·02	0·02	0·02	0·02	0·02	0·02	45	·02
90	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	0·00	45	·00
	0·00	0·67	0·73	0·78	0·84	0·90	0·97	1·04	1·11	1·19	1·28	1·38	1·48	1·60	1·73		

For Latitudes above 60° see Table (ii.), p. 28.

Azimuth 72 deg Lat 41 deg

Johnson's Explanation of his Double Chronometer Method



EXPLANATION OF THE DOUBLE CHRONOMETER RULE.

FIG (1) — C, B, A, D

FIG (2) — C, A, D, B

Let C be the true zenith of the observer, CA., CB small portions of circles of equal altitude, BD a small portion of a parallel of latitude by D.R., CD a perpendicular from C on BA, or BA produced. (Fig. (1) is for observations taken on the same side of the meridian, fig. (2) for those taken on opposite sides).*

Then the first observations worked with the D.R. lat. will place the ship at A, the second will place her at B. Therefore AB is the discrepancy (in dep.) between the two positions. Also CAD is equal to the azimuth at the first observation, and CBD is equal to that at the second.

In fig. (1)
$$\begin{aligned} AB &= BD - AD \\ &= CD\ \text{Cot. } B - CD\ \text{Cot. } A \\ &= CD\ (\text{Cot. } B - \text{Cot. } A) \end{aligned}$$

$$\therefore CD = \frac{AB}{\text{Cot. } B - \text{Cot. } A} = \frac{AB\ \text{Sec. } l}{\text{Cot. } B\ \text{Sec. } l - \text{Cot. } A\ \text{Sec. } l} = \frac{\text{Diff. long.}}{\text{Cot. } B\ \text{Sec. } l - \text{Cot. } A\ \text{Sec. } l}$$

Similarly in fig. (2)
$$CD = \frac{\text{Diff. long.}}{\text{Cot. } B\ \text{Sec. } l + \text{Cot. } A\ \text{Sec. } l}$$

The values of cot. B, sec. l and cot. A sec. l are taken from Tab. II., and designated as (a) and (b). $\therefore$ Corr. for lat. $= \dfrac{\text{Diff. long.}}{(b) \mp (a)}$, taking the upper or lower sign according as the bearings are in the same, or adjacent quadrants.

* These triangles, being supposed to be very small, may be treated as plane triangles, right-angled at D.

The corrections for the two longitudes will be AD and BD expressed in diff. long.

or, AD sec. l, and BD sec. l.

But AD = CD cot. A, and BD = CD cot. B.

$\therefore$ The corrections are CD cot. A sec. l, and CD cot. B sec. l;

or, corr. for lat. $\times$ (a), and corr. for lat. $\times$ (b),

and it is evident by fig. (1) that when the observations are in the same quadrant, both corrections must be allowed in the same direction; but when in adjacent quadrants, they must be allowed in opposite directions, as in fig. (2), to make the two longitudes agree.

It will also be seen, by fig. (1), that if the sun bore S.E$^{ly.}$ and the correction for lat. were North, that for long. would be East; and by fig. (2), if the sun bore S.W$^{ly.}$ and the corr. for lat. were North, that for long. would be West.

Hence the Rule on pp. 5 and 6:—

S. E.   S. W.

N. W.   N. E.

Conclusion

AC Johnson's Double Chronometer method is almost identical to the Sumner tangent (one longitude and an azimuth) method but with one significant difference. The accurate plot is replaced with a rough sketch and calculated corrections.

Two time sights are worked, each with an assumed latitude. The azimuth of the sun is determined for each sight but instead of accurately plotting the LOPs a rough sketch is used to determine relative positions. Then calculated corrections are applied to obtain actual latitude and longitude. The position could also be determined by drawing the rough sketch to scale. All of the data needed for an accurate plot is available but is not used in this method.