**210908 - Latitude Longitude near Noon methods comparison** (*final version with update on "**WILSON 1**" and "**WILSON 2**"*)

## 1 - OBSERVATIONS AROUND MERIDIAN TRANSIT OF A CELESTIAL BODY

Over a short time span - typically less than 1 hour - "***LAN***" (*L**ocal** A**pparent** N**oon*) ***observations*** also called "***Near Noon***" ***observations*** can provide the Celestial Navigator with a quite reliable position.

Hence these "***LAN***" observations have long been studied and there is a quite important literature to cover them.

> ***Practical steps*** - The Navigator first starts recording his sights: both UT's and Heights during a time span around the estimated time a celestial Body "crosses" his own meridian. The Navigator then processes his observations to derive his position.
>
> Since most generally the Sun is the aimed Body hence these "***LAN***" or "***Near Noon***" acronyms.
>
> 99.99 % of the time ***Upper Culminations*** only are observed: heights first rise, then stay constant and eventually decrease.

## 2 - KEY POINTS TO REMEMBER

**(2.1)** - **Plotting Heights vs. UT times on a graph** yields a "***parabola type***" series of points open towards the bottom of the graph with a symmetry axis parallel to the Heights scale.

**(2.2)** - Within the limits of quality of the observations for both a ***Fixed Latitude Observer*** and a ***constant Declination Body*** such "***parabola type***" is exactly symmetrical relatively to the *Time of Body Transit (**UT tran**)*, i.e. Body crossing the Observer's meridian. Apparent Culmination ***H culm*** occurs exactly at such ***UT tran***. Hence from Body GHA and Declination at ***UT tran*** the Navigator very easily gets his own Latitude and Longitude.

**(2.3)** - Within the same observations quality limits, for a ***moving Observer*** and/or a ***changing Declination Body***, Culmination most often does not coincide with Body Transit. ***UT culm*** becomes different from ***UT tran***. Nonetheless the symmetry axis of such [new] "observation curve" remains quite parallel to the Heights scale.

For all practical purposes this "new curve" can be considered as simply "shifted" from the previous one depicted in **(2.2)** through a **most often negligible N/S translation** and through an **often quite sizeable E/W translation**.

***(2.4) - Whichever solution is subsequently carried out, all solving methods require to accurately pinpoint - both for*** **Time and Height - the top/summit of such *actual* "observation curve", i.e. both *UT culm* and *H culm***.

**(2.5)** - From the available data - i.e. ***UT culm*** and ***H culm*** - the motions effects earlier mentioned in **(2.3)** can be taken in account and corrected for, in order to recover both ***UT tran*** and ***H tran*** . The Navigator is back to an optimum configuration to derive his own Latitude and Longitude as per **(2.2)** here-above.

Such classical corrections formulae are listed **SECTION 3** here-after.

**(2.6)** - And finally to close this important list - which otherwise could easily be extended almost *ad infinitum* - the random errors standard deviation (SDEV) comes to play as follows in the final determination of the "LAN" Position:

- SDEV directly ***degrades*** Latitude accuracy by a ratio very close from 1 to 1. And:

- SDEV directly ***degrades*** Longitude accuracy by a much higher ratio: up to 5 to 1 or even 8 or 10 to 1.

- The "lower" the ***H culm***, the bigger the Longitude accuracy degradation for one given SDEV.

## 3 - CLASSICAL CORRECTIONS FROM CULMINATION TO TRANSIT

With the following abbreviations:

- ***Lat*** : Observer's Latitude
- ***Dec*** : Body Declination
- ***NS*** : Observer's North Speed in knots (negative for South speeds). And:
- ***µDec*** : Body Declination Hourly change in Arc Minutes (algebraic values too) :

***(3.1-a) - To correct from (UT culm) into (UT tran) for** UPPER CULMINATIONS :*

***(UT culm - UT tran) in seconds of time = (48 / π) * (tan Lat - tan Dec) * (µDec - NS)** - **(1a)***

***(3.1-b) - To correct from (UT culm) into (UT tran) for** Circumpolar Bodies LOWER CULMINATIONS :*

***(UT culm - UT tran) in seconds of time = (48 / π) * (tan Lat + tan Dec) * (µDec - NS)** - **(1b)***

**(3.2)** ***- Finally to correct from H culm into H tran :***

***(H culm - H tran) in arc Minutes = ½ (µDec - NS) * (UT culm - UT tran) in hours** - **(2a)***

***On a practical stand-point and in almost all literature** (see **3.3.5** here-under):*

***H culmination = H transit** - **(2b)***

**(3.3) - <u>Notes</u>**

**(3.3.1) - *Formula (1a)* can be regarded as the FUNDAMENTAL FORMULA for all classical NAL solving methods.**

***Formulae (1a) and (1b)*** are established for the Sun with a GHA rate of 15°/hour. Even so they are ***$1^{st}$ order formulae*** only with respect to Time. For this reason they can be in error by up to 15% in extreme cases even for the Sun. They can also be used for the Planets and the Stars without significant extra fine tuning required.

**(3.3.2)** - For the Moon ***(Ref 3)*** recommends increasing the results of ***(1a)*** by 7%. It seems a reasonable starting point.

**(3.3.3) - Formulae (1a) and (1b) explicitly assume the North Speed to stay constant over the full time of the Observations, a constraint governing all types of solutions methods**.

**(3.3.4) - *Formula (1b)*** has been very rarely if ever published because the opportunities for such Observations are quite infrequent (e.g. circumpolar bright Stars during sufficiently long twilights).

**(3.3.5) - *Formula (2a)*** is a ***$2^{nd}$ order formula*** with respect to Time and as such it yields results accurate to always better than 0.01'.

However its corrective terms are so small - always inferior to 0.3' - that it has been very rarely if ever published too.

**It nonetheless remains the mathematical justification for formula *(2b)* here-above.**

## 4 - METHODS TO DERIVE *UT culm* AND *H culm*

**Accurately pinpointing both *UT culm* and *H culm*** remain **essential steps** in the course of the LAN fix process.

A quick "historical review" here-after about some successive significant methods shows their advantages and drawbacks.

In **SECTION 5**, one specific example is treated with these methods for subsequent comparisons between them.

**4.0 - AN OPTIONAL EARLY DATA PROCESSING STEP: *THE FICTITIOUS NAVIGATOR AND CELESTIAL BODY METHOD*.**

**4.0.1 - Description of the Fictitious Navigator and Fictitious Celestial Body method.**

**4.0.1.1** - Let us imagine that at some specific moment during the observations process, the Real World Navigator is "overlapped" by an extra Fictitious Navigator who is to keep exactly the same E/W speed, but has no N/S Speed. While they are to "observe" at the same Times, both Navigators will stay at exactly the same Longitudes.

At this very same time, let's assume that the Real World Body is also overlapped by an extra Fictitious constant Declination Body with which keeps exactly the same GHA's as the Real World Body.

Both the Fictitious Navigator and the Fictitious Body fulfill the conditions of **Section (2.2)** here-above. Therefore: **for the Fictitious Navigator who keeps "observing" his own Fictitious Celestial Body Heights: *UT culm = UT tran*.**

At Time **(T1)** of the first observation, the Real World Height **(H1)** and the Fictitious Body Height **(H'1)** are exactly the same. From that instant they start gradually diverging from one another.

**4.0.1.2** - For the entire time-span of Observations and if we make the assumption - which is a quite valid one - that the Cosines of the Body Azimuths always remain ***almost exactly*** equal to ***Unity*** then both **the Real World Heights and the Fictitious Heights keep diverging from one another at a rate *almost exactly* equal to *(µDec -NS)*.**

Hence, from the ***Real Word Heights set*** it is possible to derive a ***Fictitious Heights set*** showing a "curve" expected to be exactly symmetrical with respect to its own ***UT culm***.

**4.0.1.3** - With this ***FICTITIOUS NAVIGATOR AND FICTITIOUS CELESTIAL BODY METHOD*** which is no more than a handy optional data pre-treatment step, the LAN is solved as follows:

***UT tran = UT culm* of the Fictitious Heights set, and *H tran = H culm* of the Real World Heights set.**

**4.0.2** - This optional Method looks quite attractive for all graph paper methods as it frees the Navigator from recourse to **Formula (1a)** since ***it immediately yields UT tran !***

*Nonetheless*, and especially when dealing with a high number of observations (e.g. **Ref 3**), it requires a meticulous tally of the ever changing corrections to the individual Heights. This weakness can be regarded as a ***rather subtle and unnecessary complication which may easily and entirely ruin the claimed simplicity of such graph paper methods***.

**4.1 - EQUAL HEIGHTS METHOD (Ref 1 & 2)**

***By the 1930's*** - and among other "Authorities" - both the French Navy (**Ref 1**) and the UK Royal Navy (**Ref 2**) published the first "LAN type" method which advocates recording **equal heights sufficiently before and after culmination, and to monitor and record maximum height**.

***Formulae (1a) and (3)*** were then altogether published in order to solve for such LAN fixes.

***UT culm*** is taken as the average of all corresponding equal heights times, and

***H culm*** is taken as equal to the maximum height recorded.

**4.1.1 - Advantages**

**4.1.1.1** - A definite simplicity of use.

**4.1.1.2** - Can be used under degraded material conditions (no electricity …). Hence it can be considered as an Emergency method to-day.

#### 4.1.2. - Drawbacks

The observation time-spans are subject to strict constraints addressed in dedicated literature, e.g. :

**4.1.2.1** - Given random observation errors, any efficient ***UT culm*** recovery Method requires some of the Real World Heights to be observed **sufficiently far away from *UT culm***, i.e. with **sufficient increasing / decreasing rates, typically 3°/hour or more**. Otherwise, if too close from culmination, or if the [Real World] ***H culm's*** are too low - typically under @ 50°/60° for most methods - **insufficient rates** eventually translate into **unacceptable geometric dilution of position (GDOP)**. Hence the Longitude errors mentioned in **(2.6)**.

**4.1.2.2** - **On the contrary**, such equal altitudes must be taken **sufficiently close from *UT culm*** so that the "Observation curve" ***odd powers*** in Time do not start becoming significant. In other words such "curve" must always be considered as acceptably symmetrical around ***UT culm***.

**4.1.2.3 - Especially with the graph paper methods, the Navigator needs to process data sufficiently symmetrical (see 4.4.2.2.2). Accordingly the higher the *H culm* the closest all observations must all gather around *UT culm*. This constraint also governs - to variable extents - all kinds of solution methods.**

**4.1.2.4** - As an adverse consequence there can be configurations when only a **reduced observation window** is available before and after ***UT culm*** during which the Navigator is to take a minimum of 3 rising/decreasing heights in order to average at least 3 pairs of observation times.

**4.1.2.5** - Within the hard constraints of **4.1.2.3** such equal Heights should - ideally - be equally spaced between them in order to best minimize the effects of random observation errors.

**4.1.2.6** - Any rising altitude must be observed altogether with a descending one of the very same value. This implies constraints on the environment including but not limited to the actual weather encountered.

### 4.2 - EARLY CALCULATOR STATISTICAL METHODS (Ref 3)

Taking advantage of the recently introduced hand held Calculators, and ***by the late 1970's*** some Pioneers (e.g. **Ref 2**) quickly came out with statistical programs permitting adequate treatments of LAN's.

#### 4.2.1 - Advantages

**4.2.1.1** - **Statistical methods offer a definite ACCURACY ADVANTAGE over graph paper methods.**

This is definitely the main reason why they have become quite successful and popular.

**4.2.1.2** - Such Statistical methods greatly alleviate some of the graph paper methods constraints here-above.

> **Constraints 4.1.2.1 and 4.1.2.2** become irrelevant. Any [not too large] unexpected gap in the observations schedule should bring no adverse consequence as long as at least one Height close to ***H culm*** can be recorded. If sufficient observations are made anytime within the time span permitted by **constraint 4.1.2.3** then the Navigator can get a quite decent LAN fix.

**4.2.1.3** -The statistical Methods offer some simplicity of use still requiring attention in the data loading sequences (caveats in **4.2.2.1** and **4.2.2.2**)

#### 4.2.2 - Drawbacks

**4.2.2.1** - Lot of attention required to key data in. About 15 keystrokes are required per one single observation, not counting environmental data. Repeatedly and carefully checking for typing errors before entering data may become utterly painstaking. This step must be performed since *garbage in, garbage out*.

**4.2.2.2** - It was often difficult if not impossible to correct for typing errors when data already entered. A solid example to run (e.g. **Ref 3**) can require up to over 400 [uninterrupted] individual keystrokes.

**4.2.2.3** - For lack of space a number of early software versions simply used **Formula (1a)** itself which prevented them from adequately processing Moon observations. Re: Note **(3.3.2)** .

**4.2.2.4** - There is no built-in real safeguard against using LAN Software outside the strict limitations of **constraint 4.1.2.3**.

**4.2.2.5 - Relying too much onto 100% computations may lead to oversights or even blunders otherwise quite visible with graph paper methods. Lack of situation awareness may quickly bring Safety concerns.**

**4.2.2.6** - Cannot be considered as an Emergency use method to rely on.

**4.3 - ORIGINAL "WILSON 1" METHOD (Ref 4) and its further "WILSON 2" IMPROVEMENT (Ref 5)**

**4.3.1 - ORIGINAL "WILSON 1"** ***(Ref 4 - Spring 1985)***

***By the mid-1980's*** Calculators were taking advantage over the graphical methods. Any new such method yet to be invented would be worth of consideration *ONLY IF* it could significantly improve the ***UT culm*** determination process. And, *lo and behold*, ***in 1985 James N. Wilson*** came up with one such graphical method to determine ***UT culm***.

Within the limitations of **4.1.2.1** and **4.1.2.2 Wilson's 1 Method** requires 3 sets of observations:

- Rising heights,
- Culminating height(s), and
- Decreasing heights

The culminating heights yield ***H culm*** through visual interpolation on a graph.

The ***rising Heights*** are drawn on a colored graph and then visually approximated by a ***straight colored line***.

***On that same graph, and using an extra and overlapping OFFSET Time scale*** the ***decreasing Heights*** are drawn ***in different color*** and visually approximated too by a ***straight line*** so that both the rising line and the descending line intersect one another about halfway on the graph. They both cut at a given **Intersection Height**.

The Sun Height reaches this same **Intersection Height value** - ***whichever it may be*** - at a **Time T1** on the **ascending Heights branch** and at a **Time T2** on the **descending Heights branch**. Like for the previous **Equal Heights Method** (in **Section 4.1**): ***UT culm* = ½ * (T1 + T2)**.This Method shows a very clever use of overlapping OFFSET Time scales.

A nicely devised **Abacus** (***Appendix 1***) enables to visually derive the value of ***K = (48 / π) * (tan Lat -/+ tan Dec)***.

With ***K*** and ***Formula (1a/1b)*** the Navigator computes (***UT tran - UT culm).***With ***Formula (3)*** he computes ***H tran***.

**4.3.2 - IMPROVED "WILSON 2" METHOD** ***(Ref 5 - March 25, 2009)***

And - cherry on the cake - ***James N. Wilson*** did it again and subsequently came up with a very **clever improvement to his "Wilson 1" Method. His "Wilson 2" Method does not even require the Classical formula (1a)**.

**4.3.2.1** - This **"Wilson 2" Method** is very simple and it also does not show either the type of weakness mentioned in **Section 4.0.2**.

**4.3.2.3** - Back to a result established in **Section 4.0.1.2**: ***the Real World and the Fictitious Heights keep diverging overtime from one another at a rate equal to (μDec - NS).*** This is our ***1st key*** to solve this **"Wilson 2" Method**.

**4.3.2.4** - On the other hand: **over short periods of elapsed time** (e.g. a few minutes) **we may safely ignore the effect of (*μDec - NS*).** This assumption is strictly equivalent to considering that **during such reasonably short periods, we may "force" the Fictitious Heights slopes into having exactly the same values as their Real World Heights slopes counterparts**. Such is our ***2nd key*** to solve this **"Wilson 2" Method** (See **Example 5.7.2**).

## 4.4 - ENHANCED CALCULATORS / COMPUTERS STATISTICAL METHODS

***In the past decades***, significant improvements have been made onto Navigation Computers / Calculators. Both the hardware and the software of LAN programs have been significantly improved and have become more User friendly.

Reliable and accurate **Long Term Almanacs** are now routinely part of these software and are sourced onto the very best recent developments by mainly the ***French Bureau des Longitudes*** (***VSOP xxxx*** , ***ELP-xxxxx*** Solar/Planetary and Lunar analytical theories) and the ***JPL*** from the USA (***DExxx*** numerical integrations for all the Solar System Bodies).

### 4.4.1 - Advantages and drawbacks

We could now state that most of the drawbacks identified in **4.2.2.1 to 4.2.2.4** have now vanished. And for these very good reasons using modern software to solve LAN's has remained quite popular among the Community.

As for their drawbacks, we should still consider that the caveats addressed in **4.2.2.5 and 4.2.2.6** do remain relevant.

### 4.4.2 - Software Contributors

Among the many software Publishers a special mention is given to two highly respected Contributors to NavList who have made their own Navigation Software - including their LAN Software - available to the Community:

- ***Andres Ruiz González from the Kingdom of Spain***
  - *https://sites.google.com/site/navigationalalgorithms/about*
  - *navigationalalgorithms@gmail.com* . And:

- ***Peter Hakel from the USA***
  - *http://www.navigation-spreadsheets.com/*

Their LAN Software belongs to this Enhanced Software category with their many useful refinements such as nice displays of the observations set and results. Such enhancements start bringing back the **Navigator's situation awareness** much closer to the real world. Hence both Software greatly avoid the trap mentioned in **4.2.2.5** .

Their LAN Software is believed to use a refined regression Curve ($2^{nd}$ order to Time) in which the exact Body GHA rates are accurately computed, which makes them fully valid for any celestial Body, including the Moon.

### 4.4.3 - Higher order Regressions with respect to Time *(June 1989)*

**4.4.3.1 -** One way to alleviate the inconvenient of **constraint 4.1.2.3** is to implement Higher order Regressions with respect to Time so as to enlarge the "time-window" during which observations can validly be made around ***UT culm***. Such methods can also ***easily accommodate significantly lower H culm's*** - as low as 20° - vs. a minimum of 50° or even 60° generally accepted for some earlier LAN methods (e.g. 70° for the **Equal Heights Method** in **Section 4.1**).

**4.4.3.2 -** However, even if assuming that all changing elements (Observer's Latitude and Longitude, Body GHA and Declination) vary strictly linearly with respect to Time - *i.e. all second order derivatives strictly equal to zero* - ***High order Regressions Methods are to face 3 adverse environmental cumulated constraints:***

**4.4.3.2.1 -** With respect to the Time powers the **number of terms of the relevant differential functions grows faster than the number of terms of the Pascal's Triangle**. Quickly the task becomes quite cumbersome.

**4.4.3.2.2 -** The ***even order terms*** are much more significant than the ***odd order terms***. ***Note****: this makes sense since we are dealing with rather symmetrical curves (see* ***constraint 4.1.2.3****)*. As a consequence **any significant improvement** to a given level statistical Regression **requires differentiating 2 levels deeper**.

**4.4.3.2.3 -** Successive higher order Regressions widen the "time-window" **by smaller and smaller increments**.

**In spite of such cumulative obstacles it is believed that a $6^{th}$ order Regression with respect to Time is a reasonable trade-off since it doubles the size of the valid time-window of a $2^{nd}$ order regression.**

## 4.5 - FRANK E. REED's SEMI-TRANSPARENT FOLDED PAPER SHEET METHOD (Ref 6)

***Back to graphic determinations***: given the definite improvements brought by the "**Wilson 2" Method** as described in **(Section 4.3.2)**, and ***by the late 20th Century*** one could regard as a definitely impossible task to significantly simplify further the ***UT culm*** determination process through any innovative and new 100% graphical method.

Nonetheless ***Frank E. Reed*** founder and moderator of the **NavList Forum** *http://fer3.com/arc/* invented such a clear cut improvement with his "**Folded semi-transparent sheet of paper method**" and deserves full credit for it.

***In its simplest version (not using Option 4.0)***, use a sheet of semi-transparent paper to carefully graph ***Heights vs. UT times*** for all observations. Manually draw a curve to smoothly approximate as many observations points as possible.

Gently start folding the graph alongside a parallel to the heights scale and hold its two overlapping "half pages" against ambient light. Look onto and through them for both "branches" of the curve. Do not hard-fold the paper graph yet but carefully fine tune the 2 branches positions so that they overlap one another as much as possible. Then hard-fold your sheet of paper exactly alongside one parallel to the heights scale in order to keep this best superimposition visible. The hard-folded axis indicates ***UT culm***. The corresponding Height indicates ***H culm***.

***"Wilson 1" Abacus (Appendix 1)*** and ***Formula (1a)*** then enable to solve for ***UT tran***. ***Formula (2b)*** solves for ***H tran***.

### 4.5.1 - Advantages and drawbacks

**If and when used without option (4.0), and among all the graph methods earlier mentioned the "Folded semi-transparent sheet of paper method" ranks out as the best one in terms of simplicity, ease of use and reliability.**

It is only subject - and to a lesser extent than all other methods - to the limits of **constraint 4.1.2.3.**

Also, and unlike the **Equal Altitudes** and the "**Wilson 1"** & **"Wilson 2"**methods, it can accommodate in particular:

- Reasonable gaps in the observation series. Nonetheless at least one observation near ***H culm*** still required.

- And also unequally spaced Observations with respect to Time.

***It also fully qualifies as a Back-up / Emergency method.***

## 5 - ONE EXAMPLE FOR COMPARING THE PREVIOUS METHODS

Let us review the following example submitted by ***Frank E. Reed*** to NavList.

*http://fer3.com/arc/m2.aspx/Compare-Methods-LatLon-Near-Noon-FrankReed-aug-2021-g51028*

**Date: May 22, 2022**
**Observer**: Sailing 4 kts at True heading 185° on surface. **Current** : 2 kts to true East.
**SUN sights, UT / uncorrected Altitudes:**
**1 -** 16:17:20 / 72°28.5' **2 -** 16:22:40 / 72°44.5' **3 -** 16:36:45 / 73°04.0' **4 -** 16:39:15 / 73°04.5' **5 -** 16:51:25 / 72°49.0' **6 -** 16:56:20 / 72°35.5'
**All Altitudes corrections forced to** +13.0'

## 5.1 - PRELIMINARY RESULTS

**5.1.1** - **This exercise permits crosschecking various LAN Software in a thorough and meaningful way**.

- Since all altitude corrections are imposed, all users are processing exactly the same set of observations.

- Hence any difference in the end results can be attributed to only:
  - Differences in the Ephemeris used. And/or :
  - Differences in the number of significant digits published. And/or :
  - Differences in the processing software.

**5.1.2** - Whenever applicable **extra-digits** are published here-under to permit meaningful comparisons if applicable.

**5.1.3** - **Method 5.2** here-under uses the following Ephemeris for the SUN (***VSOP 2013***).

| SUN | GHA | DEC | Distance, SD and μ Dec. |
|---|---|---|---|
| 16:00 UT | 60°49.56' | N + 20°28.18' | Distance : 1.012305 UA, |
| 17:00 UT | 75°49.51' | N + 20°28.66' | SD : 15.80' ***μDec : + 0.4862 '/h*** |

**5.1.4** - **Speed Made Good**: North Speed ***(NS) -3.9848 kt*** , Speed to the East : +1.6514 , i.e. **4.3134 kt/157.4898 °**

**5.1.5** - **(μDec - NS)** = **+ 4.4710 '/h**

**5.1.6** - From the starting data, the following **approximate values** are readily derived: **Lat = N 37.2°** and **Dec = N 20.5°**

**5.1.7** - From **5.1.6** and "**Wilson 1" Abacus** (**Appendix 1**), **K = 5.9**

**5.1.8** - From **5.1.5** and **5.1.7**: **(UT culm - UT tran) = + 26.4 s** (***Reminder***: "**Wilson1" Abacus** used here as per **5.1.7)**

**5.1.9** - From **5.1.5**, **5.1.8** and **Formula (2)**, **(H culm - H tran) = 0.0164'**. As expected in **3.3.5** this is a totally negligible difference.

## 5.2 - RESULTS OF 6th ORDER LAN SOFTWARE (see 4.4.3)

> ***UT tran = 16h37m30.7s*** *and* ***LAN fix*** *at* ***N37°11.3' / W070°12.2'***
> ***UT culm - UT tran*** *=+ 26.15 s (vs. +26.4s at* ***5.1.8****)*
> ***H culm - H tran*** *= 0.0164' (vs. 0.0164' at* ***5.1.9****)*
> ***SDEV*** *= 0.23 NM (0.8' on Lon)*
> *Method Validity time span (16h11m - 17h04m : OK)*
> ***Many Body Fix Method*** *: From the* ***LAN fix*** *and at the* ***LAN time*** *get a* ***Many Body Fix*** *at* ***N37°11.3' / W070°12.2'****, and* ***SDEV*** *= 0.22 NM.*
> *The* ***Many Body Fix*** *is only 34 meters - i.e. less than 120 ft - from the* ***LAN fix****.*
>
> ***17h Fix*** *at* ***N37°09.8' W070°11.4'***

Note : The **6th order LAN method** is expected to be the most accurate of all LAN methods addressed here.

- In particular the **LAN Fix** and the **Many Body Fix** are almost identical (less than 0.022 NM apart).
- The **SDEV's** in both cases are almost identical too: 0.22' vs. 0.23'.
- These comparisons constitute a meaningful and independent check of this LAN method.

## 5.3 - RESULT OF THE EQUAL ALTITUDES METHOD (see 4.1)

**Due to its own "Equal Heights" constraint this Method cannot support the current example.**

## 5.4 - RESULTS PUBLISHED BY PETER HAKEL (See 4.4.2)

***Peter Hakel*** readily published his own results here and as follows:

http://fer3.com/arc/m2.aspx/Compare-Methods-LatLon-Near-Noon-PeterHakel-aug-2021-g51029

***UT tran = 16h37m29s*** *and* ***LAN fix*** *at* ***N37°10' / W070°11.4'***
***UT culm - UT tran*** *= not published*
***H culm - H tran*** *= not published*
***Many Body Fix Method*** *: At the LAN time get a* ***Many Body Fix*** *at* ***N37°11' / W070°12'.***
***17h Fix*** *at* ***N37°10' W070°12'***

**These results are very close from the ones obtained in 5.2.**

**5.5 - RESULTS OBTAINED BY THE FOLDED PAPER SHEET METHOD** (See **4.5**)

These results, using the **simplest version of this method**, *(**without Option 4.0**),* were published here and as follows:

*http://fer3.com/arc/m2.aspx/Compare-Methods-LatLon-Near-Noon-Couëtte-sep-2021-g51044*

***UT culm = 16h38m00s***
*From* ***Formula 1a : UT culm - UT tran*** *= +26.3 s (vs.+26.4s at* ***5.1.8*** *and +26.15s at* ***5.2****)*
***H culm - H tran*** *= not published*
***UT tran = 16h37m34s*** *and* ***LAN fix*** *at* ***N37°11.0' / W070°11.0'***
***17h Fix*** *at* ***N37°09.5' / W070°10.9'***

**The Folder Paper sheet method works very well too. Results within half of mile from 5.2 .**

**5.7 - RESULTS OBTAINED BY THE WILSON's METHODS** (See **4.3**)

**5.7.1 - ORIGINAL "WILSON 1" METHOD (See 4.3.1)**

This example carries the minimum number of Observations for the "**Wilson 1" Method** since Observations (**1,2)** and (**5,6)** are just sufficient in number to permit defining the increasing and decreasing Heights straight lines.

**Note** : Incidentally since these both Height lines are simply passing through 2 well defined points for each, it is even possible to give results just from algebraic computations, but this is not the name of the game here.

The first result of this method was first published by ***Tony Oz*** here:

*http://fer3.com/arc/m2.aspx/Compare-Methods-LatLon-Near-Noon-TonyOz-sep-2021-g51042*

On his "**Wilson 1" paper graph** carefully drawn ***Tony Oz*** derived ***UT culm*** as the average of 16h20m56s and 16h54m56s to obtain:

***UT culm = 16h37m56.0 s***

*Algebraic result (see* ***Note*** *just above): UT culm = 16h37m56.6s*

From this result by ***Tony Oz*** and with the **"Wilson 1" Abacus (Appendix 1)** we get:

***UT culm - UT tran = + 26.4 s*** and ***UT tran = 16h37m29.6s*** *(vs. 16h37m30,7s in* ***5.2****)*

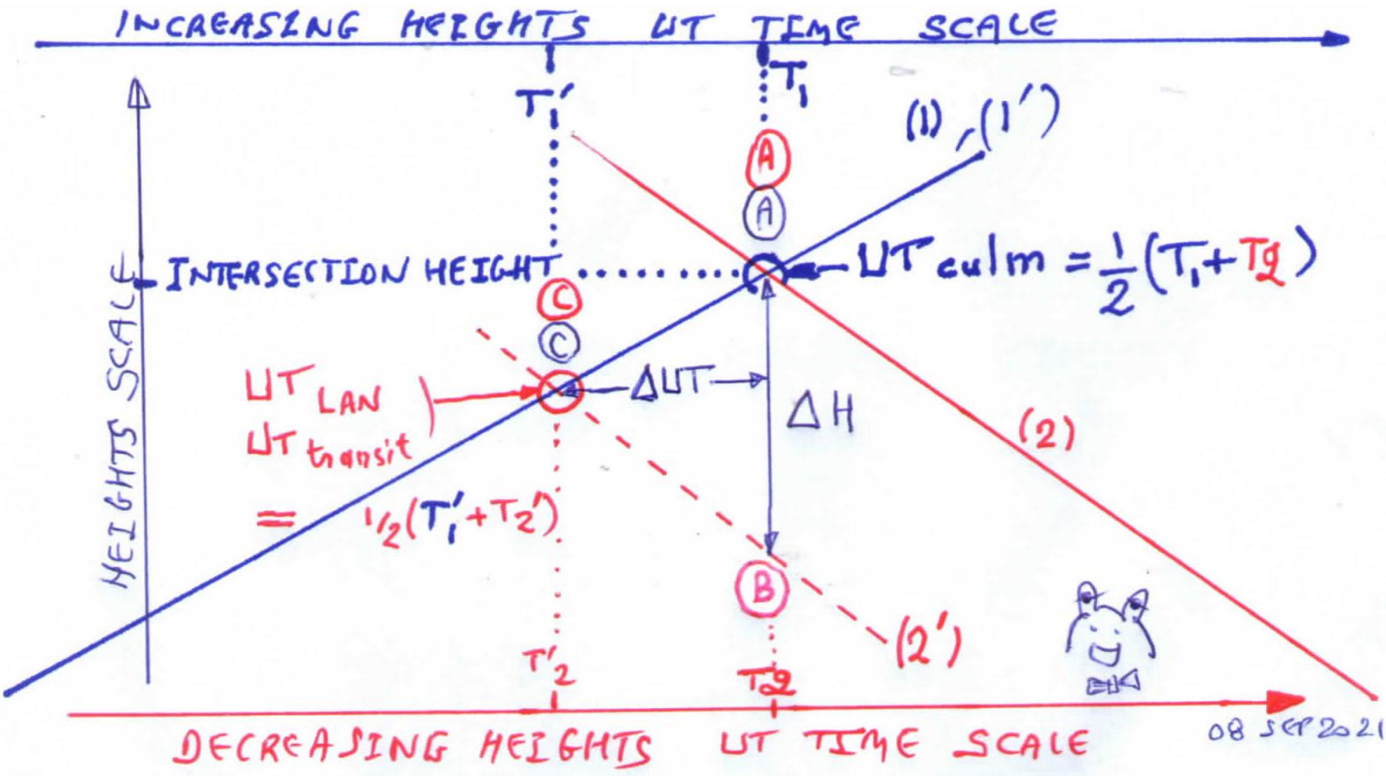
Taking the average of the published ***H culm*** values we obtain: ***H tran = 73°17.25'***, which altogether with the line here-above and the Sun Ephemeris in **5.1.3** yields the following LAN position:

***UT tran = 16h37m29.2s at fix N37°11.2'/ W 070°11.8'*** *(vs. N37°11.3' / W070°12.2' in* ***5.2****)*

**Here again, we keep getting excellent results very close from our benchmark results.**

**5.7.2 - IMPROVED "WILSON 2" METHOD (See 4.3.2)**

Starting with ***UT culm = 16h37m56.0 s*** obtained in **5.7.1** let us take a close look at our "**Wilson 1" graph**:




**Line (1)** represents the **Real World increasing Heights**. On its own **UT scale** it intersects **(2)** in Ⓐ at **T1=16h20m56s**.

**Line (2)** represents the **Real World decreasing Heights**. On its own **UT scale** it intersects **(1)** in Ⓐ at **T2=16h54m56s**.

These **identical points** Ⓐ/Ⓐ define **one same intersection Height** (*whichever it may be*).

We have seen earlier in Section **4.3.1** that ***UT culm* = ½ * (T1 + T2)**

The Time elapsed between the moments when an **increasing Height** and a **decreasing Height** pass through this same **intersection Height value** (*again: of whichever value floats your boat*) is therefore equal to **T2 - T1 = 34 m**

From **Section 4.0.1**: **let's assume that at Time T1 both the Fictitious Navigator and Sun start coming to existence.**

From **Section 4.3.2.3**, between **T1** and **T2** and due to the Observer's and Sun ***coupled N/S motion*** the **Real World decreasing Heights Line (2)** is ending up "higher" at Time **T2** than the **Fictitious decreasing Heights Line(2')**.

It shows "higher" by a **significant amount** equal to ***ΔH* = (µDec - NS) * (T2 - T1) = 4.4710 * 34/60 = 2.5 NM.**

Since it has not been subject to this Observer's and Sun ***coupled N/S motion***, at **T2** the **Fictitious decreasing Heights Line (2'**) crosses [almost] exactly point Ⓑ at 2.5 NM south of point Ⓐ.

From **Section 4.3.2.4** - During some reasonably small time span around **T2,** the slope of the **Fictitious decreasing Heights Line (2'**) can be regarded as [almost] the same as the slope of the **Real World decreasing Heights Line (2).**

Therefore the **Fictitious decreasing Heights Line (2'**) can be represented by a line parallel to the **Real World decreasing Heights Line (2)** and crossing point Ⓑ.

From **Section 4.3.2.4** again - During some reasonably small time span around **T1** the slope of the **Fictitious increasing Heights Line (1')** can be regarded as [almost] the same as the slope of the **Real World increasing Heights Line (1**). Since at **T1** they both go through point Ⓐ then we can then safely assume that **Line (1')** and **Line (1)** are identical.

Reverting to **Section 4.0.1.1**, the **Fictitious increasing Heights Line (1')** intersects the **Fictitious decreasing Heights Line (2')** at the **identical point** Ⓒ/Ⓒ at times **T'1** on **Line (1')** and **T'2** on **Line (2')**. Their mean value is at ***UT tran***.

Our final result is therefore:

**$UT\ tran = ½ * (T'1 + T'2)$**

***NOTE***: Such intersection is drawn in just a few seconds, i.e. in far less time than required for reading these lines.

From ***Tony's Oz*** graph which is meticulously drawn, we get ***ΔUT*** **= T1 -T'1 = T2 - T'2 = +26.0s**, and then obtain:

***UT tran = 16h37m30s*** *(vs. 16h37m30,7s in **5.2**)*

*Algebraic result (With UT culm = 16h37m56.6s, see **5.7.1**): UT culm - UT tran = 26,54s . Hence UT tran = 16h37m30.1s*

**Once more, we have got a *UT tran* determination very close from our benchmark results.**

### 5.7.2.1 - CHECKS AND REMARKS

**5.7.2.1.1** - The **Cosines of the Sun Azimuths** always stay between 0.96 and 1 with an average value of 0.98. Hence **assumption (4.0.1.2)** is quite valid.

**5.7.2.1.2** - Let us estimate the **consequences of neglecting (µDec - NS) over 26 s**, i.e. ***0.03 NM*** (Re: **Section 4.3.2.4**):

* **Point Ⓒ** is obtained on the **descending Line (2')** ***backwards*** from Ⓑ and it **needs a correction of 0.03 NM to the North**. **Line (2')** has a "***steeper descending slope***" than **Line (2)** in which the Real World Navigator is going southwards to the Sun which in turn is going northwards to him.
* Likewise **Point Ⓒ** is obtained on the **ascending Line (1')** ***backwards*** from Ⓐ and it also **needs the same correction of 0.03 NM to the North**. **Line (1')** has a "***less steep ascending slope***" than **Line (1)** in which, again, the Real World Navigator is going southwards to the Sun which in turn is going northwards to him.

So the new intersection of both "**corrected Line (1')**" and "**corrected Line (2')**" remains unchanged Time-wise.

Finally, neglecting **(µDec - NS)** brings ***no appreciable change*** to the Navigator's Longitude**.**


**ACKNOWLEDGMENTS AND *FINAL CONCLUSION***

Thanks to ***Frank E. Reed*** for having brought back onto the table this subject of the various LAN solving methods.

Thanks to you ***Greg Rudzinski*** for having so kindly directed my attention towards **Ref 3**.

And very special thanks to you too … ***Tony Oz*** for having unveiled to my bewildered and amazed eyes this "**Wilson 2**" Method of ***5.7.2*** !


***Could you not fall in love with such a beautiful method ???*** *You have made my day, Tony*

### *REFERENCES*


*(1) -* ***HAUTEURS CIRCUMZÉNITHALES CORRESPONDANTES***, by the **Presses de l'École Navale** *(1934)*

*(2) -* ***ADMIRALTY NAVIGATION MANUAL*** *(near page 152)*, by the **HM Stationary Office**, Vol III (*1938*)

*(3) -* ***CALCULATOR NAVIGATION*** by **Mortimer Rogoff**, by **NORTON Publishers**, ISBN 0-393-03192-6 *(May 1979)*

*(4) -* ***POSITION FROM OBSERVATION OF A SINGLE BODY***, by **James N. Wilson** in **Journal of the Institute of Navigation**, V.32 N.1 *(Spring 1985)*

*(5) -* ***THE NOON FIX***, by **James N. Wilson**, by **AUTHOR HOUSE Bloomington IN**, ISBN 978-1-4389-5866 (sc) *(25 March 2009)*

*(6) -* ***NAVLIST FORUM ARTICLE*** *by* **Frank E. Reed** *http://fer3.com/arc/m2.aspx/Latitude-Longitude-Noon-Sun-FrankReed-jun-2005-w24178 at* ***http://fer3.com/arc/***. ***Note****: This Article describes F.E. Reed's full "**Folded Paper Method**" which advocates pre-treating data as described here in **Section 4.0**. A simpler version without pre-treatment is described and demonstrated here with excellent end-results believed to be identical.*


*Au Roc Saint Luc en Pissotte, le 08 Septembre 2021 - Antoine M. "Kermit" Couëtte*

## APPENDIX 1

**Source**: ***POSITION FROM OBSERVATION OF A SINGLE BODY*** by **James N. Wilson**
**Journal of the Institute of Navigation** V.32 N.1 *(Spring 1985)*

**Determination of the quantity** ***K = (48 / π) * (tan Lat - tan Dec)*** - See **Formula 1a** in **Section 3.1-a**.

***Example***: with LAT = N 37.2 ° and Dec N 20.5° get K = 5.9 (*exact value 5.885*)

**Hint**: For the very infrequent use of **Formula (1b)** in **Section 3.1-a**: reverse the signs rule here-under.