

After allowing for typographical errors and missing explanations, Bowditch 2002 gives

Distance by Vertical Angle Measured Between Sea Horizon and Top of Object Beyond Sea Horizon

$$D = \sqrt{\left(\frac{\tan \alpha}{0.0002419}\right)^2 + \frac{H-h}{0.7349}} - \frac{\tan \alpha}{0.0002419}$$

in which D is the distance in nautical miles, α is the corrected vertical angle, H is the height of the top of the object above sea level in feet, and h is the height of eye of the observer above sea level in feet. The constants 0.0002419 and 0.7349 account for terrestrial refraction.

Setting $H = 0$ and solving for $\tan \alpha$ gives the “exact” result

$$D_s = 60 \tan^{-1} \left(\frac{h_f}{6076.1 d_s} + \frac{d_s}{8268} \right)$$

For the small angles concerned (and retaining the excess of digits) this gives the “approximate” form

$$D_s = 0.56578 \frac{h_f}{d_s} + 0.4158 d_s$$

Within the “refraction model” that scales the Earth’s radius everywhere to

$$r = \frac{10800}{\pi} \left(\frac{1}{1-\beta} \right) = 8268 \text{ nmi for } \beta = 0.1684 \text{ the exact dip short formula is}$$

$$\begin{aligned} \tan D_s &= \frac{h}{r} \csc \left(\frac{d_s}{r} \right) + \left(1 - \cos \left(\frac{d_s}{r} \right) \right) \csc \left(\frac{d_s}{r} \right) \\ &\approx \frac{h}{d_s} + \frac{d_s}{2r} + \dots \end{aligned}$$

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