Initial Great Circle Course Diagram

Robin Stuart
January 2016

The NGA Chart 17 "Great Circle Sailing Chart of the North Atlantic Ocean" has a graphical device for determining the great circle bearing of the destination from the point of departure. The instructions call for locating the point on the great circle (a straight line) at 5°, 20° or 30° longitude difference from the point of departure. The latitude of this point and that of the point of departure are then located on a pair of curves in the diagram. A line drawn through the points gives the bearing.

Assume the curves containing the latitude, L_1, of the point of departure and the latitude, L_2, of the point along the great circle track are represented parametrically as $\left(x_{L_1}, y_{L_1}\right)$ and $\left(x_{L_2}, y_{L_2}\right)$ respectively. For the line drawn between the two curves to lie in the direction of the initial great circle course, C, requires that

$$\cot C = \frac{y_{L_2} - y_{L_1}}{x_{L_2} - x_{L_1}} \tag{1.1}$$

$\cot C$ appears here as courses are measured from North rather than from the x-axis.

The great circle course, C, from an initial point with latitude, L_1, and longitude, λ_1 to a point latitude, L_2, and longitude, λ_2, can be written as

$$\tan C = \frac{\sin \Delta\lambda \cos L_2}{\cos L_1 \sin L_2 - \sin L_1 \cos L_2 \cos \Delta\lambda}$$

where $\Delta\lambda = \lambda_2 - \lambda_1$. (The numerator and denominator of the right hand side can be used as the two arguments of the arctangent function to obtain results in the correct quadrant.)

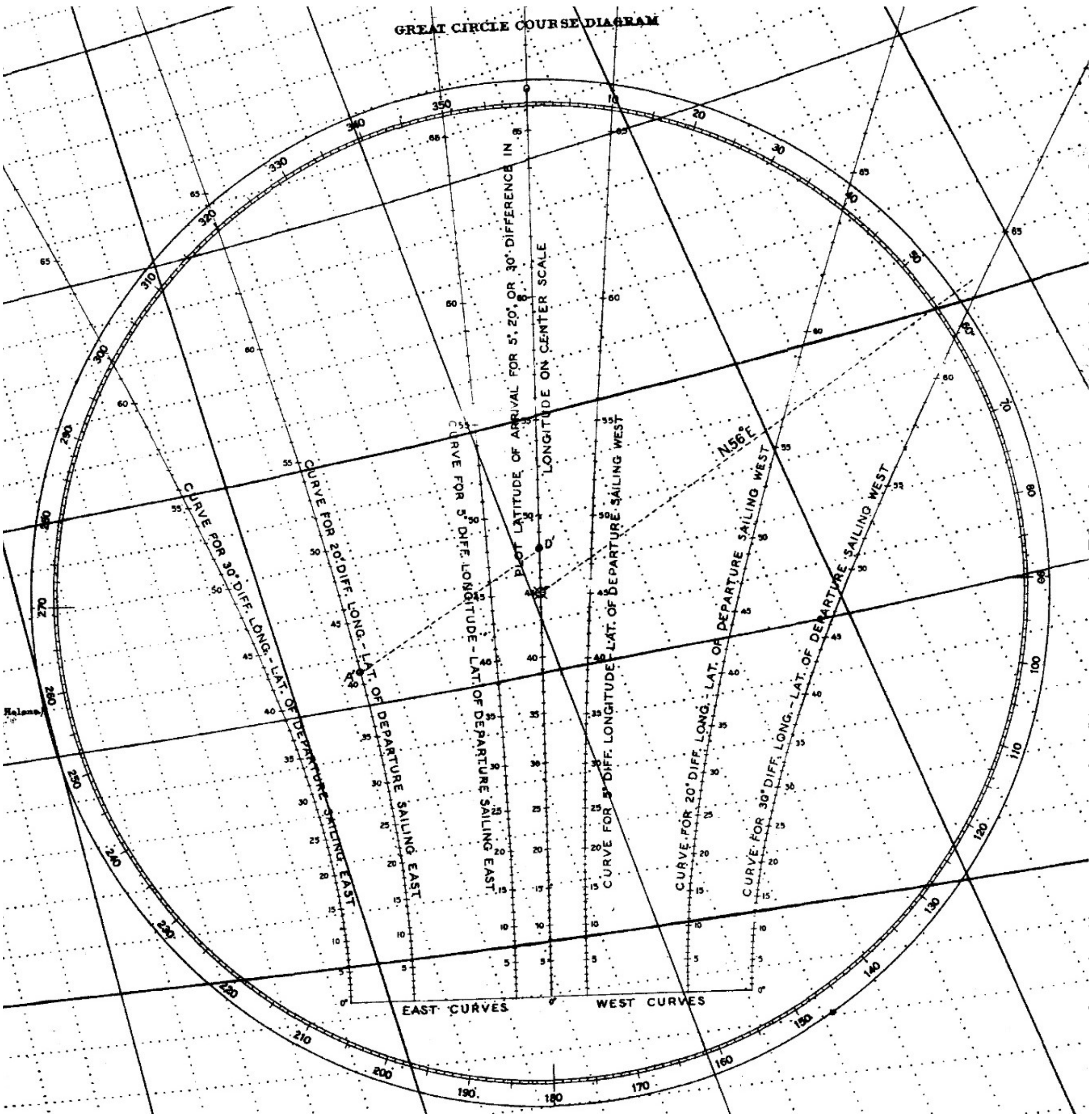
This equation can be rearranged to

$$\cot C = \frac{\tan L_2 - \cos \Delta\lambda \tan L_1}{0 - \left(-\sin \Delta\lambda \sec L_1\right)}$$

which has the same form as eq.(1.1) and it is therefore a simple matter to read of the required parametric form of the scales

$$\left(x_{L_1}, y_{L_1}\right) = \left(-\sin \Delta\lambda \sec L_1, \cos \Delta\lambda \tan L_1\right)$$

$$\left(x_{L_2}, y_{L_2}\right) = \left(0, \tan L_2\right)$$



COURSES

To determine the course (see example on chart) draw a straight line between the points of departure and destination (A B). Note the latitude (D) of a point on the track 20° of longitude from the point of departure (A), and mark it on the middle line of the COURSE DIAGRAM (D).

On the same diagram mark the latitude of the ship (A) on the east curves when sailing eastward, and on the west curves when sailing westward, and by means of a parallel ruler transfer the straight line connecting these two points (A D') to the center of the compass and read off the course (N 56° E. true.)

This gives the course for *starting* from the point of departure (A). It is necessary to change the course from time to time in order to keep on the same great circle track as will be seen by measuring the course from other points on the same track to the point of destination (B)