Navigation on Shackleton's voyage to Antarctica

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On 19 January 1915, the Imperial Trans-Antarctic Expedition, under the leadership of Sir Ernest Shackleton, became trapped in their vessel *Endurance* in the ice pack of the Weddell Sea. The subsequent ordeal and efforts that lead to the successful rescue of all expedition members are the stuff of legend and have been extensively discussed elsewhere. Prior to that time, however, the voyage had proceeded relatively uneventfully and was dutifully recorded in Captain Frank Worsley's log and work book. This provides a window into the navigational methods used in the day-to-day running of the ship by a master mariner under normal circumstances in the early twentieth century. The conclusions that can be gleaned from a careful inspection of the log book over this period are described here.

Keywords: celestial navigation, dead reckoning, double altitudes, Ernest Shackleton, Frank Worsley, Imperial Trans-Antarctic Expedition, Mercator sailing, time sight

Introduction

On 8 August 1914, the Imperial Trans-Antarctic Expedition under the leadership of Sir Ernest Shackleton set sail aboard their vessel the steam yacht (S.Y.) *Endurance* from Plymouth, England, with the goal of traversing the Antarctic continent from the Weddell to Ross Seas. *Endurance* was under the command of Captain Frank Worsley (Fig. 1).

What transpired has become an epic tale of survival from the heroic age of polar exploration. *Endurance* became trapped in the pack ice of the Weddell Sea until 27 October 1915 when she had to be abandoned before finally sinking on 21 November. After camping on the ice for 165 days, the crew reached Elephant Island in three small boats on 15 April 1916. On 24 April, Shackleton, Worsley and four others embarked on a perilous 800 nautical mile (1,500 km)

passage in the 22½ foot (6.9 m) *James Caird* to seek rescue from South Georgia. It is ultimately a tribute to Shackleton's leadership and Worsley's navigational skills that all survived their ordeal.

Captain Frank Worsley's original log books and related materials are now in the collection of Canterbury Museum. Recent papers have examined the contents of the logs in detail. Bergman et al. (2018) describe the navigational methods used and how they were applied to successfully complete the voyage of the *James Caird*. The log pages from that crucial period were transcribed and replicated with full annotation. Bergman and Stuart (2018) examined the navigational methods that needed to be employed during the long polar night while trapped in the ice pack of the Weddell Sea. Out of sight of land, lunar occultations

Figure 1. Frank Worsley c. 1910. Steffano Webb Studio, Christchurch. Canterbury Museum 2010.27.1

were used to determine Greenwich Mean Time (GMT) and rate the ship's chronometers. Stars took the place of the Sun for fixing latitude and longitude. Bergman and Stuart (2019) examined the Sun sights and their reductions as recorded in the log around 21 November 1915 to conclude that the location of the wreck of *Endurance* is further south than the position of 68°39'30"S 52°26'30"W stated in the log and is subject to uncertainties of several nautical miles in both latitude and longitude.

The remainder of the log book, recorded during the period of normal operation from 8 August 1914 to 19 January 1915 when the *Endurance* became trapped in pack ice, is by comparison unremarkable. It does, however, provide an engaging and important historical window into the real day-to-day operation of the vessel and the navigational techniques as they were actually used on a vessel under the command of a master mariner in the early part of the twentieth century. From the entries it can be

deduced how on-board clocks were adjusted as the vessel moved west and what procedures were followed in obtaining time sights for longitude.

Celestial navigation guided *Endurance* from Plymouth via Madeira to Tenerife, through Cape Verde to Buenos Aires, then further to South Georgia and on to Antarctica. Many of the methods that Worsley employed are not those used or taught today. The standard noon sights and time sights have been described in detail elsewhere (Bergman et al. 2018), but during this period of normal operation additional techniques occasionally make an appearance. These are described here.

The section 'Formulas and Observations' describes the collection of navigational formulas, positions of key landmarks and hydrographic observations from earlier Antarctic expeditions that Worsley chose to place on page 1 of his log book. The section 'Passage to the Antarctic' gives a general overview of the observations recorded in the log during the early stages of the expedition. 'Celestial Navigation' briefly summarises the types of astronomical observations that were made during this part of the voyage. The section entitled 'Time Management Aboard Ship' deals with how clocks were adjusted as the vessel shifted in longitude and the procedures followed for taking time sights. 'The Ship's Log' describes types of patent logs used to track the distance sailed. The section 'Double Altitudes' describes a method used on a few occasions whereby two observations taken at different times are combined to determine both latitude and longitude. 'Mercator Sailing' describes how the course and distance to waypoints were calculated during the ocean passages.

Formulas and Observations

Page 1 of log book (Fig. 2) declares:
> *Lieut F.A. Worsley R.N.R.*
> *commanding s.y. Endurance*
> *on a Voyage of discovery to the Antarctic*
> *Work Book*

This is followed by a collection of formulas, positions of landmarks and observations from

previous Antarctic expeditions of the sea ice conditions they had encountered. The nautical chart symbols for "Rock with less than 6^{ft} on at LW [Low Water]" and "[Rock] awash at LW" are also noted. A statute mile is 5,280 feet (1,609 m) and an Admiralty nautical mile (NM) is 6,080 feet (1,853 m) in length. The handy distance conversions, " kts = 5 mls - 53 ft." and "13 kts = 15 mls" are given in which "mls" indicates statute miles, or 5,280 feet, and "kts" denotes Admiralty nautical miles (NM).[1] The denomination "mls" is, however, used for nautical miles in the rest of the log book.

Like any navigator of the time, Worsley carried copies of the Nautical Almanac covering the expected period of the voyage and other "navigation books". These included an epitome that contained the methods, formulas and tables required for sight reduction and course and distance calculations. Table 1 shows the collection of formulas considered to be important and that were collected on the first page of the log book. The right hand column gives the algebraic expression to be evaluated and the left hand column gives the form as written in the log.

Distance to visible horizon in nautical miles	
Dist: horizon = sq: root height in feet + $\frac{1}{7}$ in miles	$D = 1.15\sqrt{h_f}$ $\approx \sqrt{h_f} + \frac{1}{7}\sqrt{h_f}$
Distance off in nautical miles	
Height in feet × .565 ÷ angle in ′ = dist in miles	$D = \left(\dfrac{10{,}800}{6{,}080\,\pi}\right)\dfrac{H}{\alpha}$
Sailings	
Mercator	
Log D.Long – Log M.D.L = Tan: Co.	$\tan C = \text{D.Lon.}/m$
Sec Co. + Log D.Lat. = Log Dist.	$D = \text{D.Lat.} \times \sec C$
Parallel	
Sec:Lat + Log Dep = Log D.Long	$D = \text{Dep.}$
Cos.Lat + Log D.Long. = Log Dep.	$\quad = \text{D.Lon.} \times \cos L$
Log Dep. – Log D.Long = Cos:Lat	
Middle Latitude	
Cos:M.Lat + Log D.Long – Log D.Lat = Tan:Co.	$\tan C = \dfrac{\text{D.Lon.} \times \cos M}{\text{D.Lat.}}$
Sec:Co + Log D.Lat = Log.Dist.	$D = \text{D.Lat.} \times \sec C$
Longitude	
✶'s W^{ly} Hour Angle + ✶'s R.A = R.A.M. (S.T. P II)	$\text{LST} = \text{LHA} + \text{R.A.}$
R.A.M. – Acc^{D} M.S.R.A. = M.T.S.	$\text{LMT} = \text{LST} - (\text{GST} - \text{GMT})$
Meridian Passage	
✶'s RA – SRA. (Precedg:Noon) = Aprox Time − Acceleration = M.T. ✶'s Mer. Pass	

Table 1. Formulas as they are written in the log book (left) and their algebraic equivalents (right).
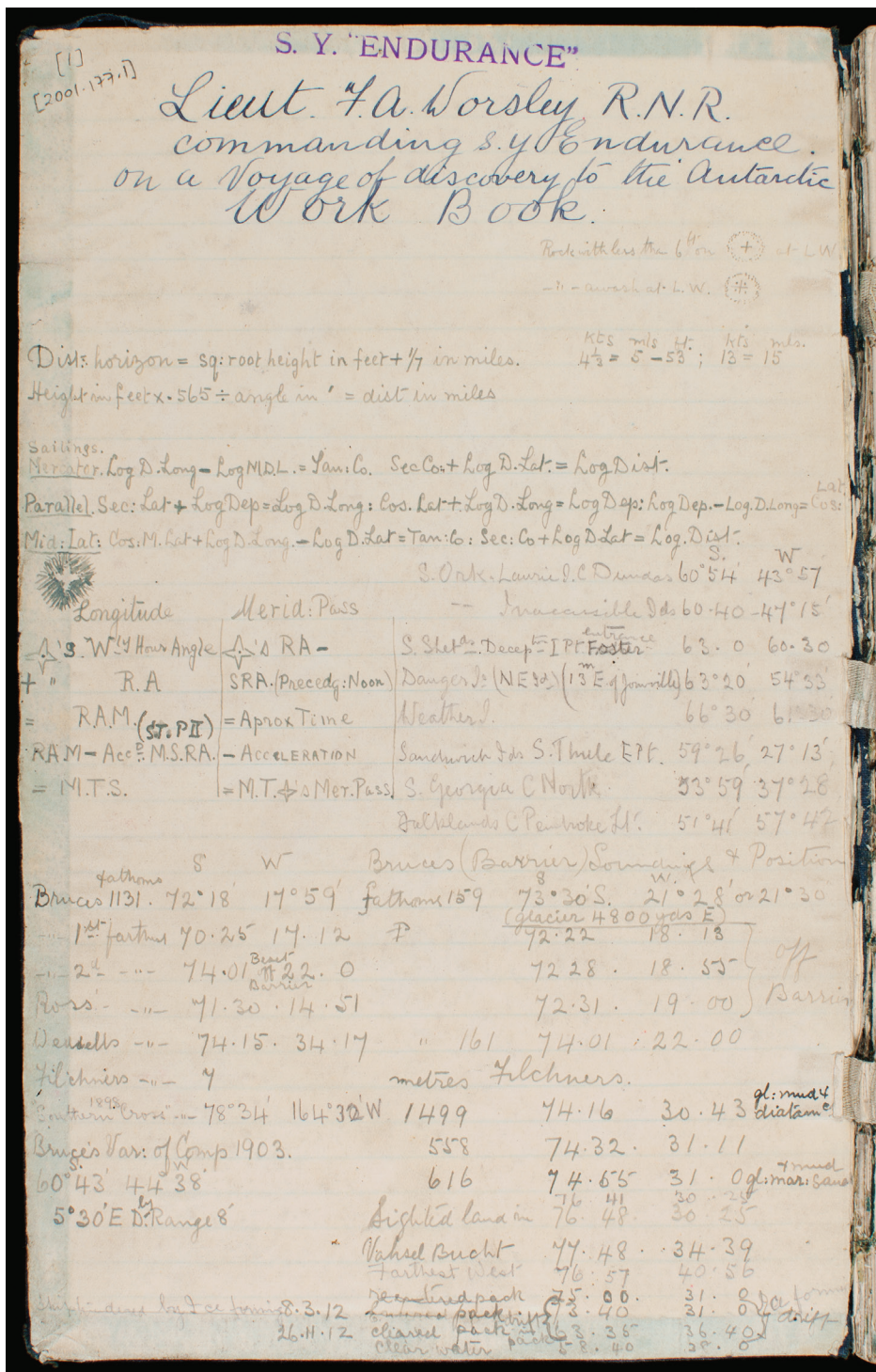
Figure 2. First page of Captain Frank Worsley's log and workbook giving a collection of navigational formulas, positions of key landmarks and hydrographic observations from earlier Antarctic expeditions. Canterbury Museum 2001.177.1

Unlike the noon sight and time sight reduction calculations (Bergman et al. 2018) the formulas listed might be needed from time to time but would not necessarily be used on a daily basis.

Distance to the visible horizon in nautical miles

This is the distance on the Earth's surface from the observer to the visible horizon. Conversely it is also the geographic range of visibility at which an object of known height may be seen by an observer at sea level. The height of the observer's eye in feet is denoted h_f. The distance of the horizon is primarily determined by the Earth's curvature but atmospheric refraction increases the distance over what is obtained from purely geometric considerations. The multiplicative factor of 1.15 may vary depending on the refraction model being used and is the one given in the *Admiralty Manual of Navigation* (1914: 64 Art.57.). The formula as written in the log is not analytically correct but may represent a rule of thumb that is considered 'close enough' that can be quickly evaluated without the need for multiplication or it may be shorthand for the correct formula.

The distance, D, to a lighthouse with a focal height of H_f feet above sea level that has just appeared on, or disappeared below, the horizon can be found by applying the formula to h_f and H_f and summing the results,

$$D = 1.15\sqrt{H_f} + 1.15\sqrt{h_f}$$

The height of the focal plane of the light is taken from published Light Lists or nautical charts.

Distance off in nautical miles

Using a sextant to measure the apparent angular height, α, of an object of known height, H_f feet, allows its distance off to be determined. The formula represents the application of simple proportions to relate the angle subtended by an object of known size to its distance from the observer under the approximation of small angles. The multiplicative factor, $10,800/\pi$ converts radians to minutes of arc and, as noted

previously, 6,080 is the number of feet in an Admiralty mile.

The formula can be used to determine the ship's distance from a coastal feature such as a lighthouse when its full extent is visible and its height is accurately known.

The formula does not apply for objects lying beyond the visible horizon. Worsley attempted to use it to determine the distance of Mount Percy on Joinville Island from Patience Camp on the Weddell Sea ice but rejected the result as being far too large (Bergman and Stuart 2018: 87).

The Sailings

The path of shortest distance between two points on the Earth's surface is the arc of a great circle, however, the course or direction to be sailed relative to true north varies continuously over the track making them somewhat complicated to follow. Mariners therefore favour a rhumb line or Mercator sailing in which the course remains fixed. Moreover, in many cases the extra distance involved is not significant. In practice over short distances, certain approximations can be applied which simplify and streamline the calculations. Each approximation defines its own particular 'sailing'.

The formulas are written in terms of logarithms, which is how they would have been evaluated in practice (Bergman et al. 2018: 27). In Worsley's formulas, Cos:, Sec:, Tan: is actually shorthand for the logarithms of these trigonometric functions that would have been extracted directly from tables. Other quantities appearing in the formulas, D.Lat., D.Lon., Dep., C, D, have been defined and discussed elsewhere (Bergman et al. 2018: 28).

Mercator Sailing

This is a path of constant course, C, between the point of departure and the destination and is plotted as a straight line on a standard Mercator projection chart. Such a chart has the property that it is conformal or angle-preserving, and hence bearings can be measured directly from it. In the log the full Mercator sailing calculation

is used to compute the course and distance over long tracks between waypoints while crossing the Atlantic to Buenos Aires, to South Georgia and on to the Antarctic.

M.D.L. stands for 'meridional difference of latitude' also known as the difference in meridional parts and sometimes denoted by *m*. The meridional part is proportional to the distance that a given parallel of latitude lies from the equator under a Mercator projection and its value in nautical miles would be obtained from tables in standard navigational texts of the time. For a spherical Earth, the meridional part for a given latitude, *L*, is

$$\mathrm{MP}\left(L\right)=\frac{10{,}800}{\pi}\ln\tan\left(45^{\circ}+\frac{L}{2}\right) \qquad (1)$$

where the factor $10{,}800/\pi$ is the Earth's radius in nautical miles which are taken to subtend a minute of arc on the Earth's surface. The M.D.L. for a track from latitude, L_1, to latitude, L_2, is M.D.L. = MP(L_2) – MP(L_1).

The layout of calculations as they appear in the log is shown in Table 4.

Parallel Sailing
This is sailing due east or west along a parallel of latitude. It is useful if longitude cannot be reliably determined and was widely used when making landfall.

Middle Latitude Sailing
The scale of the Mercator projection varies with latitude but over relatively small distances that scale can be assumed constant. Middle latitude sailing uses the constant longitude scale factor derived from the average of the initial and final latitudes, hence the name. It is explained in detail in Bergman et al. (2018: 28).

The Middle Latitude formulas themselves are seldom used. They are seen in the log only in the calculation of course and distance between Elephant Island and South Georgia on 24 April 1916 just prior to setting off on the famous voyage of the *James Caird*. At other times, such as computing dead reckoning (DR) positions, traverse tables (Bergman et al. 2018:29) were used.

Longitude
Longitude by time sight of the Sun was a procedure that was performed at least once daily, weather permitting. The reduction procedure would be ingrained in the navigator's memory and needed no special entry in this crib sheet of formulas. Longitude by time sight of a star or planet was done less often and the steps required are carefully set out. Examples of reductions of this type can be found in Bergman and Stuart (2018: 74) and the explanation there is given in terms of sidereal time, which is the natural time scale to adopt for sight reductions of stars and planets. The nomenclature that Worsley adopts was standard in navigation (Brown 1904) but to a modern reader may seem rather baroque.

R.A.M.
  = Right Ascension (R.A.) of the Meridian
  = Local Sidereal Time (LST)

M.S.R.A.
  = Mean Sun's R.A. at Greenwich Noon
  = Greenwich Sidereal Time (GST) at Greenwich Noon

Acc[D] M.S.R.A.
  = accelerated Mean Sun's R.A.
  = GST
  = GMT + Acceleration + M.S.R.A.

M.T.S = Mean Time at Ship

The annotation (S. T. PII) indicates that the quantity M.S.R.A. can be found in the column labelled "Sidereal Time" on page II of the monthly pages in the Nautical Almanac (1914). In the above, Acceleration = 0.002738 × GMT, which accounts for the sidereal day being just $23^h56^m4.1^s$ long and hence sidereal time advances at a faster rate than standard solar time. Acceleration is obtained from tables by separately looking up the contributions for GMT hours, minutes and seconds and adding them together (Bergman and Stuart 2018: table 3).

In the above formulas, Greenwich Mean Time (GMT) is 0^h at noon which was the standard for nautical time keeping up to and including 1924.

The final formula gives an estimate of when a particular star or planet crosses the meridian and can therefore be used in a sight to fix latitude.

Only approximate times are initially required as ex-meridian corrections can be applied (Bergman and Stuart 2018: 73). When the exact time of meridian passage is required the acceleration is subtracted from the approximate result. In these formulas M.S.R.A. is abbreviated to SRA.

Soundings, magnetic compass variation and other observations made by earlier expeditions are given under the formulas. The referenced expeditions are:

Weddell: James Weddell 1823–1824
Ross: James Clark Ross 1839–1843
Southern Cross: Carsten Borchgrevink, Southern Cross Expedition 1898–1900
Bruce: William Speirs Bruce, Scottish National Antarctic Expedition 1902–1904
Filchner: Wilhelm Filchner, Second German Antarctic Expedition 1911–1913

In addition it is known that Worsley had a copy of Nordenskjöld and Andersson (1905) but was dubious about the accuracy of its charts (Bergman et al. 2018: 33).

A position of 77˚48'S 34˚39'W is given for Vahsel Bucht, which is German for Vahsel Bay on the eastern edge of the Weddell Sea and was where Shackleton had planned to make a landing (Shackleton 1920).

Passage to the Antarctic

The Imperial Trans-Antarctic Expedition departed London's South West India Dock on 1 August 1914 (Worsley 1915: 3). After calling at Southend and adjusting the compasses, further stops were made at Margate and Eastbourne before arriving at Plymouth on 5 August. At noon on 8 August they departed Plymouth to begin the passage to Antarctica via Madeira, the Canary Islands, Buenos Aires and South Georgia.

Page 3 of the log records the shipboard events up to 12 August along with the ship's draft, weather conditions, the various species of starfish, sea urchins and bivalves trawled from the bottom, and sightings of humpback whales.

Left S.W.India Dks London 9·30 a.m. Aug: 1ˢᵗ 1914
Draft F 12'·9". A. 13'.11" Called at Southend adjusted
compasses.   Margate   &   Eastbourne. Proceeded Plymouth on
4ᵗʰ Aug. 5·30 P.M. Arr: Plym.ʰ 6 P.M. 5ᵗʰ Aug.
Left Plymouth Noon Aug: 8ᵗʰ Sir Ernest landed in Admirals
barge, ⚓'d Cawsand Thick weather & S.W. gale. 9ᵗʰ Aug 4 a.m. pro-
ceeded on voyage (Draft F. 11'.7" A.13'.6") against mod. SW gale.
2·30 P.M. Lizard N.3'. Put ship on port tack set sail. 7·53 Wolf
                                                    log 87
Rk. S.E. 5' 10·25 Took depʳ St. Marys ^ bg N48W.14' set co.S42˚ W. Error
18˚ W.
Aug: 10ᵗʰ light Nˡʸ airs High WS.W. swell.
10·35a.m. 48˚.28' N. 6˚56'W. Surf. Temp. sample water & speed net
5.P.M. Shot Otter trawl in 70 fms. 180 fms wire. Trawled
about 3ᵐ & hauled in about 80fms (Echinus acutus, Polyzoa (cellapora)
Ophiothrix fragilis, Ophiocoma nigra, Luidia ciliaris, Pinna rudis
Specimens similar to Echinus acutus ground E. of Rame Hd.

Aug 11ᵗʰ 7.5PM Surf. Temp & full speed townet diatom sample
Several blowers (Megaplera) seen.

Aug 12ᵗʰ 10·25A.M. Surf temp, townets, coarse, medium fine & very fine.

At 2.30 pm, the Lizard Lighthouse was sighted to the north and its distance determined by measuring its height as observed by sextant and applying the "Distance off in nautical miles" formula discussed in the previous section. At 7.53 pm, this procedure was repeated for Wolf's Rock Lighthouse when it lay 5 NM to

the southeast.

Worsley took departure off St. Mary's from the Isles of Scilly on 10.25 pm on 9 August 1914 and records the ship's log as reading 87 NM suggesting that it was probably set to zero at the anchorage in Cawsand Bay off Plymouth. The distance to Peninnis Lighthouse on St Mary's would have been found by noting the height of the eye when it disappeared below the horizon. Adding the geographic range of visibility of the lighthouse to the distance to the observer's horizon, both calculated using the "Distance of the visible horizon" formula, yielded 14 NM. The focal height of the Peninnis light is 118 feet (36 m) and working backward it is possible to determine the height of eye, h_p, that was being used in the calculation. The analytically correct "Distance of the visible horizon" formula in the form that it is given in the right hand column of Table 1 yields a height of eye above sea level, h_f, of around 2 feet (0.6 m), which from the deck of *Endurance* is unrealistically low. Applying the formula exactly as it is written in the log and in the left hand column of Table 1 gives h_f anywhere in the range of 6–11 feet (1.8–3.4 m), which is much more reasonable and suggests that this was the formula that Worsley was actually using in practice.

Inspection of the ship's log readings and course steered indicate that the bearing of N48˚W is magnetic. The quoted 18˚W error is the difference from the ship's compass reading from true and is dominated by the magnetic variation or declination. Residual deviation due to imperfections in the compass adjustment process had been reduced to within ½˚.

Worsley lists the lengths of the various legs of the voyage on the second page of the log (Fig. 3). Also recorded is the time period over which each leg was undertaken and the actual distance sailed. The amount of coal remaining at key stages of the journey is also recorded.

The waypoints that Worsley used to chart a course for *Endurance* along with the periods over which they appear in the log are listed in Table 2. Their names are given in the form in which they can be found in the log and may

differ from the modern spelling or designation but in such cases the stated position allows the present day counterpart to be identified. Along with the location of the waypoint itself, the ship's position and date it first appears in the log is also listed. This information is used in plotting the rhumb lines in Figure 4.

In some cases different names are used for the same general location within the log page in Figure 3. These may be different again from the name used to refer to the waypoint in Table 2 based on the log entries. Thus Madeira/Funchal/Fora Island all refer to the same general location as do Tenerife/Santa Cruz in the Canary Islands and Saint Vincent/Bird Rock in Cape Verde. "Salvages" refers to a small group of islands between Madeira and the Canary Islands.

On 20 August 1914, at a distance of 120 NM from the Port Fora Islet waypoint for Madeira, double altitude sights and a latitude by Polaris are taken along with depth soundings.

There are no log entries for the period 21–25 August 1914 while the ship was in Madeira, but from Figure 3 it is seen that on 24 August *Endurance* made a short, 54 NM, side trip from Madeira to the Desertas Islands to the southeast and back to the Funchal anchorage, "Funchal ⚓ age 32˚38'N 16˚54'W" (Worsley 1915: 12). The expedition departed Madeira on the 25 August.

The next entry is on 26 August and is a double altitude sight giving a position about 50 NM north of the Salvages Islands and about 150 NM from Tenerife.

They departed the Canary Islands on 30 August and passed by Cape Verde on 5 September. No sights are recorded for that day but the noon position is given as 16˚52'N 25˚5'W. This is consistent with the account that the quartermaster gives in his diary, "At noon we passed St. Vincent Island, Cape Verde Is. passing about 100 yds. off the mouth of the harbour" (Orde-Lees 1916).

On 7 September, two days out from Cape Verde, Worsley calculates the distance between future waypoints and between the lightships or lightvessels, bearing the designation "L.V.", that would be encountered off the coast of

South America.

After passing Cape Verde, *Endurance* headed for Ilha da Trindade until 18 September, when the course was changed for "30′ off C.S. Thomé". Trindade was passed far to the north on 21 September.

The log entry for 29 September contains a calculation of the distance from Buenos Aires to Port Stanley in the Falkland Islands but this track was never used.

On 3 October, at an observed latitude of 31°39′ S, a "Long by Soun^{dg}" (longitude by sounding) is given as 50°27′ W. Both quantities are double underlined indicating they are the result of observation. The position is about 32 NM off the coast of southern Brazil. Although the depth is not recorded at the stated position it would have been around 55 fathoms (100 m).

Endurance stopped in Buenos Aires from 9 to 26 October during which time the ship's chronometers were rated. This would later prove crucial as, during the long period of entrapment in the Weddell Sea ice pack, it provided the only means of keeping track of their longitude until a

series of observations of lunar occultation were made (Bergman and Stuart 2018). The exact method used to rate *Endurance's* chronometers is not recorded in the log, however, it is known (Cifuentes-Cárdenas and Nicodemo 2009) that the Naval Observatory did provide a time signal by means of a timeball atop the Central Office of Hydrography that was dropped at 13 hours LMT for Argentina's official meridian through Córdoba. A radiotelegraph time service had also been available since 1912.

The expedition left Grytviken, South Georgia, on 5 December 1914 and in the log Worsley computes the course from Cooper Island lying at 54°46′S 35°32′W off the south eastern tip of South Georgia to Southern Thule. On 11 December, the log notes "Pack ice" for the first time. The information recorded around that time is limited to course, distance and position and a few brief notes, as indicated following the entry for 7 December 1914, when Worsley was, "Bitten separating dogs, unable to take observations for 3 weeks. Hudson & Greenstreet doing the navigation observations". No further

Figure 3. Legs as sailed on the voyage of *Endurance* from London to Antarctica along with the amount of coal remaining at various stages. Canterbury Museum 2001.177.1

Figure 4. Mercator chart plotting daily noon positions track of *Endurance* in black with a selection of dates labelled for reference. Rhumb line courses to waypoints that were computed and followed along the way are plotted in red. Waypoints and stops are labelled and shown as red dots.

Waypoint	Location	Date	Ship's Position		Waypoint Position	
			Latitude	Longitude	Latitude	Longitude
Pt. Fora. I. Lt.	Madeira	10-Aug	49 ° 47 ′ N	6 ° 0 ′ W	32 ° 43 ′ N	16 ° 39 ′ W
Bird Rk.	Cape Verde	31-Aug	26 ° 14 ′ N	18 ° 0 ′ W	16 ° 55 ′ N	25 ° 1 ′ W
Trinidad I.	off Brazil	6-Sep	14 ° 28 ′ N	25 ° 44 ′ W	20 ° 30 ′ S	29 ° 22 ′ W
C. S. Thomé	Brazil	18-Sep	7 ° 22 ′ S	28 ° 25 ′ W	22 ° 10 ′ S	40 ° 30 ′ W
C. Frio	Brazil	28-Sep	22 ° 39 ′ S	40 ° 48 ′ W	23 ° 2 ′ S	41 ° 59 ′ W
C. S. Maria	Uruguay	29-Sep	24 ° 33 ′ S	42 ° 41 ′ W	34 ° 40 ′ S	54 ° 0 ′ W
Lobos I.	Uruguay	5-Oct	33 ° 28 ′ S	51 ° 52 ′ W	35 ° 2 ′ S	54 ° 53 ′ W
C. Saunders	South Georgia	28-Oct	38 ° 1 ′ S	55 ° 3 ′ W	54 ° 4 ′ S	36 ° 32 ′ W
S. Thule	Southern Thule	5-Dec	54 ° 46 ′ S	35 ° 32 ′ W	59 ° 26 ′ S	27 ° 45 ′ W

Table 2. Waypoints listed in the log and used in the passage to Antarctica giving the date and ship's position when they first appear.

observations or reduction calculations appear until 28 December.

The 'Barrier' or Antarctic ice shelf was sighted to the southeast at 4.30 pm on 10 January 1915 and sketches were made. In the following days, skirting the barrier, a number of soundings were made and the nature of the material brought up from the bottom is described in some detail. The log entry for 19 January 1915 records:

Fast in pack. Course. S71˚W. 23m.
To Vahsel Bucht S31˚W. 85m.

From then on *Endurance* would remain in the grip of the ice until being crushed and finally sinking on 21 November 1915.

Figure 4 shows the daily noon positions and track of *Endurance* in black during its voyage into the South Atlantic Ocean. Rhumb lines that were followed in the course of the journey are shown in red. Waypoints and ports of call are labelled and marked as red dots. On 9 September 1914, the track turns sharply east under the influence of the Equatorial Counter Current.

Celestial Navigation

During the voyage navigational sights were taken almost exclusively of the Sun and the first of these appears in the log on 10 August 1914. Along the way a few sights were also taken of stars and the planet Venus.

A sight for latitude by measuring the altitude of the pole star, Polaris, was taken on the evening of 20 August approaching Madeira and yielded 33˚54′N.

On 13 September, a round of sights of γ Draconis for latitude (Worsley 1915: 22) and Venus (Worsley 1915: 26) for longitude were taken while in the mid-Atlantic just north of the equator and may have been for practice. Despite its Bayer designation γ Draconis is the brightest star in the constellation of Draco and at magnitude 2.24 is quite prominent. It is one of today's 57 official navigational stars and referred to by the name Eltanin in the *Nautical Almanac*.

On 4 October, roughly a day's run off the coast of South America, a time sight was taken of Venus.

As could be expected, from the entries of the log, it is evident that noon sights of the Sun for latitude and A.M. time sights for longitude were the norm while underway. In the proximity of land, star sights, depth soundings and double altitudes were also used. Far from land no P.M. time sight was generally taken unless the A.M. sight could not be made.

Time Management Aboard Ship

As a ship travels east or west its effective time changes and the clocks must be regularly adjusted
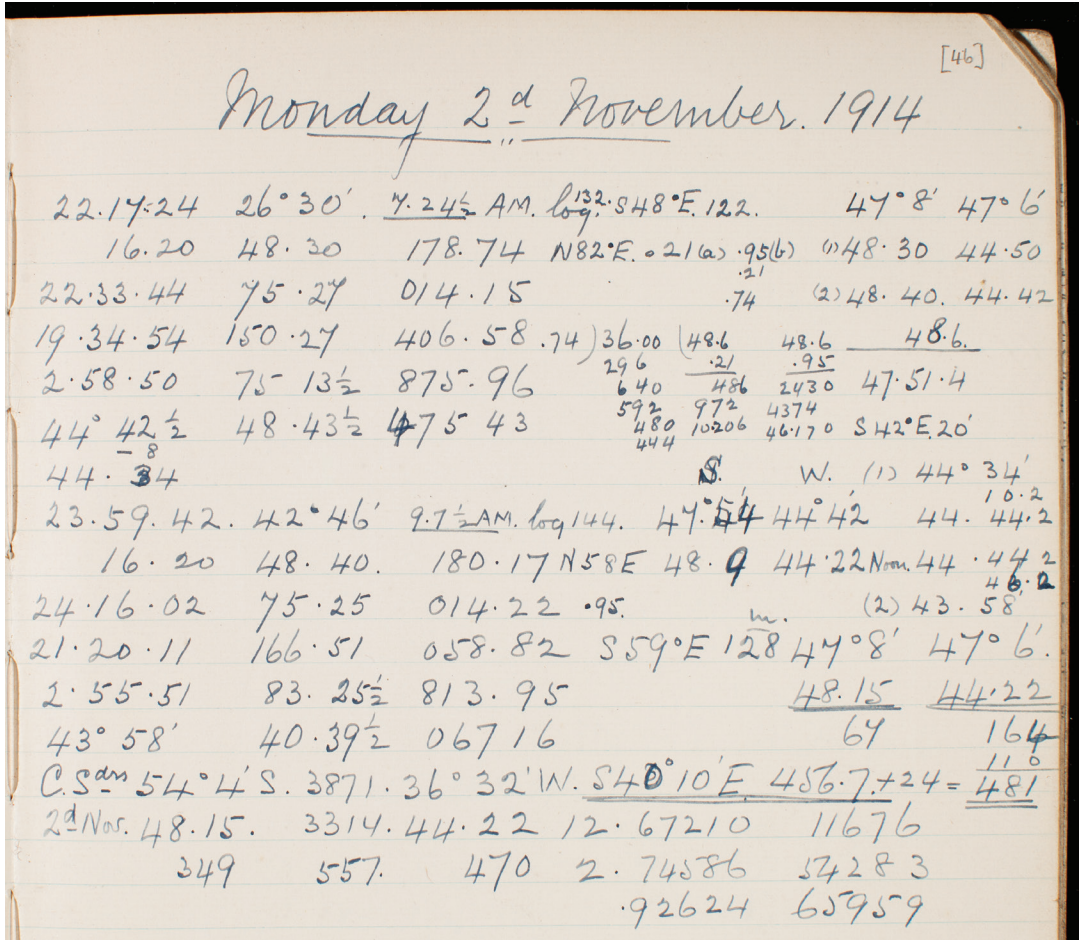
Figure 5. Log entry for 2 November 1914 showing a pair of time sights of the Sun that are combined as a double altitude fix for latitude and longitude. The last few lines at the bottom of the page show the Mercator Sailing calculation of the course and distance to Cape Saunders on South Georgia. Canterbury Museum 2001.177.1

to keep shipboard activities synchronized with the daylight hours. In contrast, the chronometers, of which there were 24 on board *Endurance* (Worsley 1998: 101), maintained GMT. They would only be very rarely reset and careful records were kept of their individual chronometer errors and rates. Most Sun sights in the log record both GMT and Ship's Time (ST). For example, the first time sight in the log entry for 2 November 1914 (Fig. 5) records GMT $22^h17^m24^s$ and ST $7^h24\frac{1}{2}^m$. Comparing GMT with ST through the voyage gives insight into the procedure used to adjust the latter. It is evident that Zone Time (ZT) with integer

hour offset from GMT was not kept on board the *Endurance*. As ST is given to the minute and sometimes to half or even a quarter minute, it is obvious that ST did have significance.

In earlier times it was customary to set ST to 12 hours at Local Apparent Noon (LAN). Thus, when observing the noon sun for latitude, "8 bells" were struck when the Sun started to descend. At that time the ST was set to 12 hours. This management style was not appreciated in the galley, as the noon meal may not be ready in time when sailing eastwards, or risked getting cold when sailing westwards (Petersson 1973).

LAN does not necessarily exactly coincide

with the time when the Sun's altitude is at maximum due to its changing declination and the north-south component of a ship's speed (Wilson 1985) and this practice was therefore becoming less favoured as ship speed increased. A later practice was to change ST in the morning, aiming at getting ST = 12 hours at LAN. Then the noon sight observation would be made at exactly ST = 12 hours, irrespective of whether the altitude was rising or setting. Generally, the altitude is nearly constant for several minutes around noon and latitude obtained from it is relatively insensitive to the exact timing. This practice is described in *Admiralty Manual of Navigation* (1914: 93), but was replaced a few years later when *Admiralty Manual of Navigation* (1922: 295–297) prescribed ZT to be used as ST, a practice still used today.

The standard reduction procedure that Worsley used for time sights to obtain longitude (Bergman et al. 2018: 27) requires the observer's latitude, *L*, as input. In principle then the observed longitude obtained depends on how well *L* can be estimated. However, when the body being observed is on the 'prime vertical' (i.e. due east or west) the resulting longitude is completely insensitive to the value used for *L*. The log entries give enough information to compute the azimuth of the body when the time sight was taken and it is clear that some considerable effort went into ensuring they were made as close to the prime vertical as possible. At certain times of the year the Sun is very low in the sky or below the horizon when on the prime vertical. At such times taking the time sight as far as possible from the meridian minimises the sensitivity to the estimated latitude.

If the Sun is close to the prime vertical, the A.M or P.M. time sights yield the local apparent time (LAT) directly. A comparison with the ST noted for the sight immediately gave the actual error of the ship's clock. An estimate of the expected change in longitude up to noon was made and the ship's clock adjusted accordingly. This estimate was easy enough for steam ships able to keep constant course and speed, but often more of a guesswork for navigators of sailing ships.

The LAT expressed in astronomical time gives the local hour angle (LHA). In this period it was customary to also express the LHA as a time in terms of hours, minutes and seconds. It is a simple matter to pre-calculate the hour angle when a body is at the prime vertical, through the relation, cos LHA = cot *L* tan *δ*, where *δ* is its declination.

Making a correction for estimated longitude change since the previous noon gave the ST for the A.M. Sun sight.

Astronomical LAT is, if less than 12 hours, equivalent to civil time P.M., otherwise astronomical LAT plus 12 hours is equivalent to civil time A.M.

The Ship's Log

The ship's distance travelled through water is read from the patent or taffrail log. During the passage from UK to Antarctica, log readings are noted from time to time. At least two patent logs were carried, one by Thomas Walker & Son Ltd, Birmingham, UK, and another by John Bliss & Co., Inc., New York. When both logs were streamed they were distinguished by Walker and Bliss, or just W and B. On certain days the log reading noted at the A.M. sight corresponds to the run from the previous noon, indicating that the log was reset to zero at noon. This is, however, not shown generally, and the notations of log readings are too sparse to allow any conclusions to be drawn on log management in general.

Double Altitudes

On some occasions it may prove to be impossible to take a sight around noon on a given day or more frequent updates to the observed position may be required such as when the ship is close to land. If two time sights are made separated in time by at least an hour and a half or two, then both the latitude and longitude at the time of the second sight can be determined. This requires that the DR course and distance is carefully tracked between the two sights. The

'Double Altitude Method' was used on 20 and 26 August, 7 September, 2 and 4 November and 7 December. In all cases the method was applied in relatively close proximity to land where accurate knowledge of position was crucial. A double altitude sight also appears on 28 December when Worsley was able to resume taking sights after recovering from being bitten by dogs.

The term double altitude covers a number of computational methods that either combine sights of two distinct objects made at the same time or two sights of the same object, typically the Sun, made at different times. The method that Worsley applied is described in Johnson (1909) where it is called the "Double Chronometer Method". Johnson's *On finding the latitude and longitude in cloudy weather and at other times* was first published in 1889 and on its title page bears the pronouncement that the look up tables contained therein are "(Supplied to H. M. Ships by Admiralty Order)". At its heart the method closely mirrors a standard running fix that a modern navigator might perform by plotting a pair of lines of position (LoP) on a chart or plotting sheet and advancing the earlier one to account for the run of the ship. The vessel's position at the time of the second sight is where the two lines cross. The difference is that in Johnson's method the fix is obtained by calculation alone and no plotting is required.

Suppose a time sight is taken. The latitude L_0 is used in the sight reduction to find the longitude, λ_0. The Sun's azimuth at the time of sight is Z_1 and is recorded. Over a period of a few hours the ship's run is noted and the position L_0, λ_0 is advanced to a new DR position L_2, λ_1. A second time sight is taken and observation is reduced using the DR latitude, L_2, to obtain the observed longitude, λ_2. The Sun's azimuth is Z_2.

The positions L_2, λ_1 and L_2, λ_2 lie on lines of position, also known as Sumner lines, running in directions $Z_1 \pm 90°$ and $Z_2 \pm 90°$. Johnson's method finds the intersection of these lines. The required position of the ship, L, λ, satisfies the equations

$$\frac{(\lambda - \lambda_1)\cos L_2}{L - L_2} = \tan(Z_1 \pm 90°) \qquad (2)$$

or equivalently

$$\frac{\lambda - \lambda_1}{L - L_2} = -\sec L_2 \cot Z_1 \triangleq m_1 \qquad (3)$$

Similarly

$$\frac{\lambda - \lambda_2}{L - L_2} = -\sec L_2 \cot Z_2 \triangleq m_2 \qquad (4)$$

The solution of the simultaneous equations (3) and (4) can be written in the form

$$\lambda = \lambda_1 - m_1 \left\{ \frac{\lambda_2 - \lambda_1}{m_2 - m_1} \right\} \qquad (5a)$$

$$= \lambda_2 - m_2 \left\{ \frac{\lambda_2 - \lambda_1}{m_2 - m_1} \right\} \qquad (5b)$$

$$L = L_2 - \left\{ \frac{\lambda_2 - \lambda_1}{m_2 - m_1} \right\} \qquad (6)$$

Note that the two different arrangements of the expression for λ provide a check on the calculation.

The quantities m_1 and m_2 absent their signs are denoted (a) and (b) respectively by Johnson and are extracted from his Table II. As was common practice in navigational calculations, (a) and (b) are taken to be positive and rules are given to recover the information contained in their signs. The signs of the corrections for longitude is deduced from the requirement that Equations (5a) and (5b) yield the same value for λ. Combining this information with the bearing at which the sight was taken allows the sign of the latitude correction, the quantity in curly brackets, to be determined. As the sign of the cotangent function alternates around the four quadrants, the rules in this case could get rather involved. However, a fairly simple mnemonic device is provided to facilitate this. It requires writing down the letters corresponding to the cardinal directions, 'N. E. S. W.' in an order dictated by the particular circumstances of the sights and connecting two of them by a diagonal

line. This device does not appear in the log entry for 2 November 1914 (Fig. 5) but can be seen in the first double altitude calculation on 20 August and again on 28 December. It is illustrated by example in the double altitude reduction demonstrated later in this section.

The required azimuths, Z_1 and Z_2, are taken from the tables in Johnson (1909) or elsewhere such as ABC tables (Lecky 1912).

Figure 6 shows the LoPs for the sights of 2 November 1914 as they might be plotted by a modern navigator. The dotted line is the LoP from the first sight before being advanced to intersect with the second. The red 'X' is the position that Worsley obtained by calculation. The correction obtained for the DR latitude for this particular sight is around 50′ and is much larger than typically seen.

Table 3 replicates the double altitude sight reduction for 2 November 1914 following the
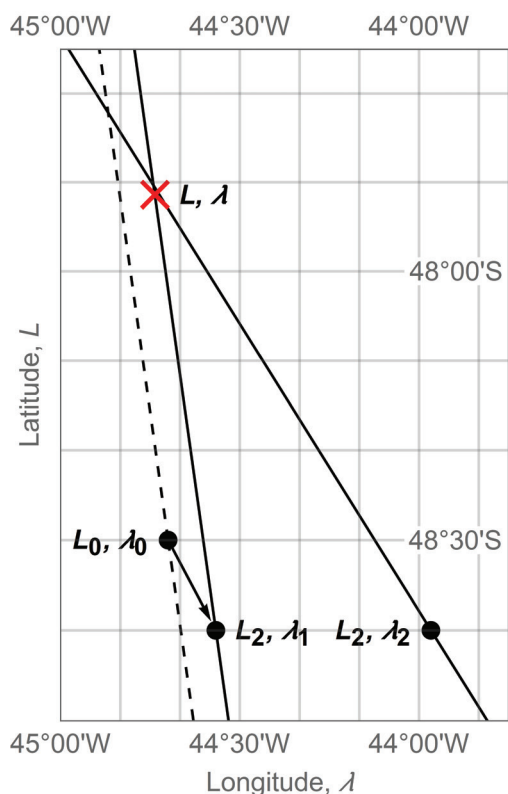

Figure 6. Plot of the Lines of Position used in the double altitude fix on 2 November 1914.

conventions set out above in which signs for the intermediate quantities have been retained. The calculations of these corrections appear on the log book page in Figure 5 to the right of the first time sight. The calculation of the correction for latitude is performed using long division, which is then used in long multiplication to obtain the longitude corrections. The reduction of the time sights that go into this calculation occupy the left hand side of the log page (Fig. 5) and a full explanation of these calculations can be found in Bergman et al. (2018). A demonstration of the method and notation that Worsley employed in carrying out the long division can be found in Raper (1840: 2–3). In that convention the quotient is written to the right of the '(' symbol. The position obtained from the reduction of double altitude sights is quite insensitive to the initial estimate of latitude, L_0.

In the calculation in Table 3, the signs of the corrections coming from Equations (5) and (6) have been kept for the convenience of the modern reader. Worsley would have followed the steps laid out by Johnson (1909). Noting that both sights were taken when the Sun was in the northeast quadrant he would conclude that the corrections to be applied to λ_1 and λ_2 in Equations (5a) and (5b) have the same name. They are therefore either both E or both W and only W produces the equal values longitude, λ.

To find the name of the latitude correction calls for taking either the first or second sight and writing down the quadrant in which the sight was taken. In the present case this would be 'N. E.' for both. Under these the complementary letters 'S. W.' are written. In this block of four letters a diagonal line is drawn from the name of the longitude correction, W, to find the name of the latitude correction as shown below.

Quadrant
N.	E.
S.	W.	longitude correction
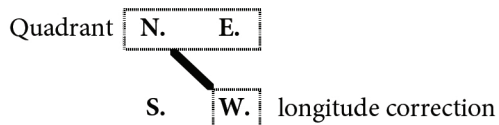
The name of the latitude correction is therefore N. It can be seen that these procedures produce the same outcome as carrying signs

First Time Sight			
Latitude (L_0)	48 °	30 '	S
Longitude (λ_0)	44 °	42 '	W
Azimuth (Z_1)	N	82 °	E
Run	10 S	8 E	
DR Latitude (L_2)	48 °	40 '	S
DR Longitude (λ_1)	44 °	34 '	W

Second Time Sight					
DR Latitude (L_2)	48 °	40 '	S	m_1 :	-0.21
Longitude (λ_2)	43 °	58 '	W	m_2 :	-0.95
Azimuth (Z_2)	N	58 °	E	$m_2 - m_1$:	-0.74
				$\lambda_2 - \lambda_1$:	36 '

Position				Corrections	
Latitude (L)	47 °	51.4 '	S	$\leftarrow L_2 +$	48.6 '
Longitude (λ)	44 °	44.2 '	W	$\leftarrow \lambda_1 -$	10.2 '
Longitude (λ)	44 °	44.2 '	W	$\leftarrow \lambda_2 -$	46.2 '

Table 3. Double altitude reduction for the time sights made on 2 November 1914.

	Latitude	Merid. Parts	Longitude		
Destination	L_2	MP(L_2)	λ_2	$\log_{10}$ (D.Lon.)	$\log_{10}$ (sec C)
Position	L_1	MP(L_1)	λ_1	$\log_{10}$ (M.D.L.)	$\log_{10}$ (D.Lat.)
	D.Lat.	**M.D.L.**	**D.Lon.**	$\log_{10}$ (tan C)	$\log_{10} D$
	$60 \times (L_2 - L_1)$	MP(L_2)–MP(L_1)	$60 \times (\lambda_2 - \lambda_1)$		

Table 4. Layout for Mercator Sailing calculations found in the log. The factor of 60 in the last line of the table converts degrees to nautical miles.

2 November 1914	Latitude	Merid. Parts	Longitude		
Cape Saunders	54 ° 4 ' S	3871	36 ° 32 ' W	2.67210	0.11675
Current Position	48 ° 15 ' S	3314	44 ° 22 ' W	2.74586	2.54283
	D.Lat.	**M.D.L.**	**D.Lon.**	9.92624	2.65958
	349	557	470		

Course, *C*   S 40 ° 9 ' E

Distance, *D*   456.6   NM

Table 5. Replicated Mercator Sailing calculation to Cape Saunders on 2 November 1914.

through the calculation and does not depend on whether it is applied to the first or second sight.

Mercator Sailing

Mercator Sailing calculations correctly account for the changing scale of the Mercator projection as a function of latitude and therefore produce accurate results when the distance to the destination is relatively large. They are seen in the log whenever the course and distance to the current waypoint are being computed and always follow the same standard layout as shown in Table 4. As noted earlier, DR calculations relied on Middle Latitude sailing by means of traverse tables.

An example of a Mercator Sailing calculation is shown at the bottom of the log entry for 2 November 1914 while underway from Buenos Aires to Grytviken. The destination is Cape Saunders, which lies at the northern entry to Leith Harbour on South Georgia.

This Mercator sailing calculation is replicated in Table 5. Here, and in other such calculations in the log, Worsley does not follow a consistent strategy for recording the integral part, or characteristic, of logarithms. They are nevertheless correctly handled in the overall calculation. In Table 4, an increment of 10 has been added to the value whenever a negative logarithm is encountered, which was standard procedure for the time, as described by Bergman et al. (2018: 27), and avoids the need to perform subtraction.

Summary and Conclusions

Captain Frank Worsley's log book from the vessel S.Y. *Endurance* has been examined in detail with the aim of understanding the navigational methods that were applied in practice in the running of a ship under the command of a master mariner in the early part of the twentieth century.

Coastal piloting off the southern coast of England relied on sextant measurements of the angular heights of lighthouses to determine distance off. Departure was taken from the Peninnis Lighthouse on the Isles of Scilly by noting the moment that the light disappeared below the horizon, which gave its distance. Courses and distances to intermediate destinations and waypoints were computed based on Mercator sailings. Over the shorter distances required for dead reckoning of the ship's daily run, middle latitude sailing calculations, obtained from traverse tables, were applied.

Underway when relatively far from land, only a morning Sun time sight for longitude and noon sight for latitude were taken, weather permitting. Approaching 100 NM of land or other hazard, additional time sights and other navigational methods were employed. At such times double altitude sights that gave both latitude and longitude were made. An observation of longitude by sounding was made when approaching the coast of South America. A few sights are recorded using stars or planets but these were not the norm.

By computing the Sun's azimuth from the log entries, it is clear that time sights were normally made when the Sun was very close to the prime vertical. There, the longitude obtained is independent of the observer's assumed latitude. Time sights could also be made at other azimuths when the proximity to land required it or the altitude of the Sun was too low on the prime vertical.

The ship's clock, and therefore the time at which daily routines were performed aboard, was adjusted daily as *Endurance* sailed west. It appears that the clock was set so that 12 hours coincided with local apparent noon at the ship's expected position.

The log contains comprehensive lists of the positions of lighthouses and lightships in the vicinity of the ports of call.

Endnote

1 "kts" is an abbreviation for knots which is more accurately a speed in nautical miles per hour.

References

Admiralty Manual of Navigation. 1914. London: His Majesty's Stationery Office. Retrieved from https://archive.org/details/manualofnavigati00grea

Admiralty Manual of Navigation: Volume 1. 1922. London: His Majesty's Stationery Office.

Bergman L, Huxtable G, Morris BR, Stuart RG. 2018. Navigation of the *James Caird* on the Shackleton Expedition. *Records of the Canterbury Museum* 32: 23–66.

Bergman L, Stuart RG. 2018. Navigation of the Shackleton Expedition on the Weddell Sea Pack Ice. *Records of the Canterbury Museum* 32: 67–98.

Bergman L, Stuart RG. 2019. On the location of Shackleton's vessel *Endurance*. *Journal of Navigation* 72: 257–268.

Brown. 1904. *Brown's Star Atlas.* Glasgow: James Brown & Son. Retrieved from: https://hdl.handle.net/2027/coo1.ark:/13960/t9377z76w

Cifuentes-Cárdenas A, Nicodemo C. 2009. Historia del Observatorio Naval Buenos Aires. *Asociación Argentina de Astronomía* Book Series 2: 297–306. Retrieved from https://archive.org/details/HistoriaDeLaAstronomiaArgentina

Johnson AC. 1909. *On Finding the Latitude and Longitude in Cloudy Weather and at Other Times.* 32nd ed. London: J.D. Potter. Retrieved from https://hdl.handle.net/2027/uc1.$b72143

Lecky STS. 1912. *Wrinkles in Practical Navigation.* London: George Philip & Son, Ltd. Retrieved from https://hdl.handle.net/2027/mdp.39076005029280

Nautical Almanac. 1914. *The Nautical Almanac and Astronomical Ephemeris for the Year 1914.* London: The Lords Commissioners of the Admiralty. Retrieved from https://hdl.handle.net/2027/njp.32101043286465

Nordenskjöld NOG, Andersson JG. 1905. *Antarctica: Or Two Years Amongst the Ice of the South Pole.* London: The Macmillan Company. Retrieved from http://catalog.hathitrust.org/Record/001877086

Orde-Lees T. 1916. Diary, 19 August 1914 & 5 September 1914. Rauner Stefansson Library, MSS 185, Dartmouth College. Retrieved from https://sites.dartmouth.edu/library/2016/07/15/shackletons-endurance-expedition-a-crewmans-view-5/

Petersson BC. 1973. Oceannavigering under segel. *Longitude* 8: 21–43.

Raper H. 1840. T*he Practice of Navigation and Nautical Astronomy.* London: R. B. Bate. Retrieved from https://books.google.com/books?id=g30PAAAAYAAJ

Shackleton EH. 1920. *South: The Story of Shackleton's Last Expedition.* London: The Macmillan Company. Retrieved from https://archive.org/details/cu31924032382529

Wilson JN. 1985. Position from observation of a single body. *Navigation - Journal of the Institute of Navigation* 32: 85–95.

Worsley FA. 1915. Workbook of S. Y. *Endurance* from 1 August 1914 to 30 August 1915. Canterbury Museum, 2001.177.1.

Worsley FA. 1998. *Shackleton's Boat Journey.* London, New York: W. W. Norton & Company Inc.